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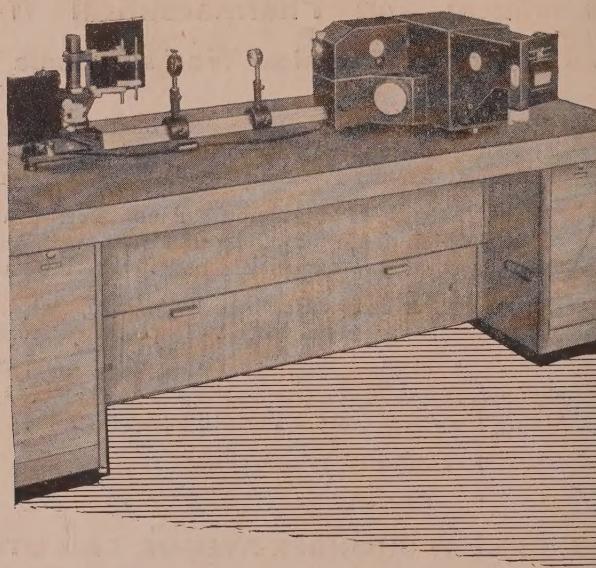
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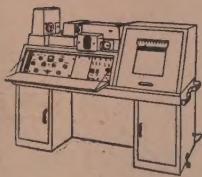
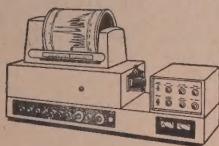
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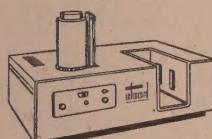
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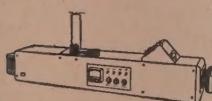
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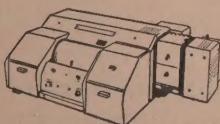
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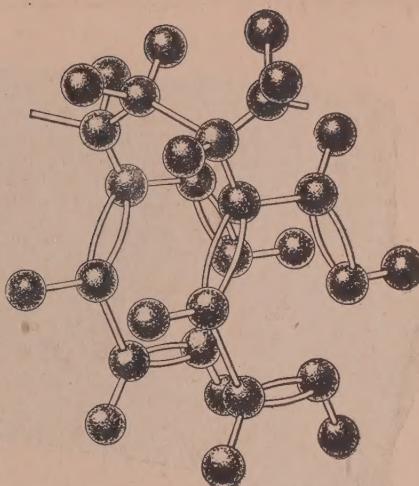
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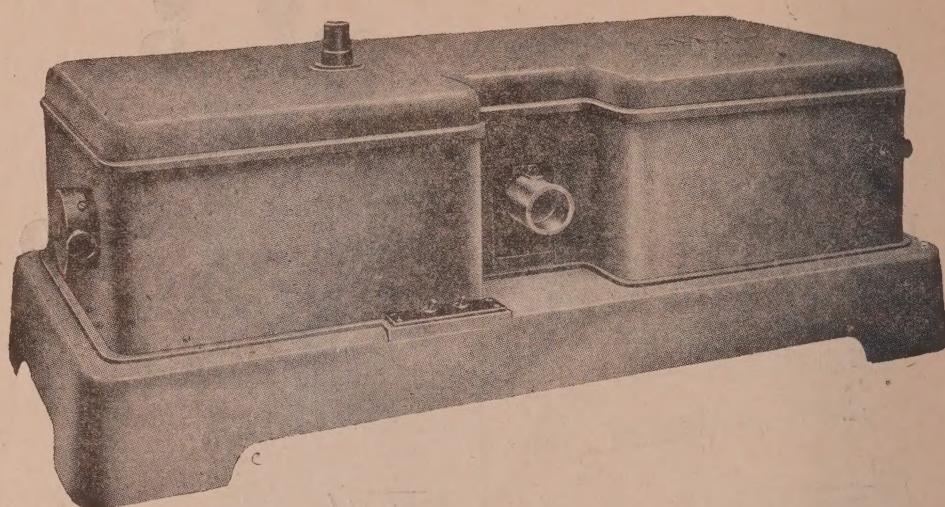
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PENETRATION FACTOR IN ALPHA-DECAY

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(Received, April 4, 1960)

ABSTRACT. In this paper we have calculated the penetration probability of α -particle through a potential barrier by using the Lanczos method of solving the Schrödinger equation near the nuclear boundary. We have chosen the Woods-Saxon potential for the nuclear field. The value of the half-life of $^{214}_{84} Po$ calculated from this penetration factor comes out to be twice the experimental value.

INTRODUCTION

The various calculations of the penetration factor in the process of α -decay that have been made till recently according to one-body model, give different results which indicate that there is room for finding out a reasonably accurate approximation method. As for the potential for such calculations we have used the Woods-Saxons' diffuse potential model near the nuclear surface, since we feel it approaches reality more than others; outside the nuclear potential Coulomb field predominates. The calculation of the wave function in the region where only Coulomb potential is present has been made by the Riccati method as given by Abramowitz (1949) (cf. Froberg, 1955). So we get an accurate solution of the Schrödinger equation for the Coulomb region. In solving the Schrödinger equation in the neighbourhood of the nuclear boundary we have employed, instead of the WKB approximation, the method of solution given by Lanczos (1938), according to which the differential equation has been equated to an error term which is proportional to Tshebysheff's polynomial of a given order. Consequently the equation admits of a finite power series solution. The error term vanishes at the zero points of the Tshebysheff's polynomial and thus an interpolating power series solution has been obtained. In this case the error term is of an oscillatory character, and the maximum error at any point of the range is less than that of the Taylor's series solution with the same number of terms.

The penetration factor has been calculated from the value of the wave function at the point near the nuclear boundary where the potential energy is equal to the kinetic energy of the emitted α -particle ($r = r_2$).

We have next estimated the half-life from the calculated value of the penetration factor.

MATHEMATICAL FORMULATION

The equation for u , which is r times the wave function of the radial part of the Schrödinger equation can be written as

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2} [E - W(r)]u = 0 \quad \dots \quad (1)$$

where $W(r)$ for $l = 0$ takes the form as

$$W(r) = U(r) + V(r).$$

$$\text{where } U(r) = \frac{2(Z - 2)e^2}{r} \quad \text{for } r > r_1$$

$$= U(r_1) = \text{constant} \quad \text{for } r < r_1,$$

$$\text{and } V(r) = - \frac{V_0}{1 + e^{\frac{r-R}{a}}} \quad \text{for } r < r_1$$

$$= 0, \quad \text{for } r > r_1.$$

r_1 is taken as the point where the nuclear potential drops to $-\frac{V_0}{100}$. For $r < r_1$,

the Coulomb potential is assumed to have a constant value.

To solve equation (1) in the region $R \leq r \leq r_1$ we write it in the form

$$\frac{d^2u}{dx^2} + \left\{ \frac{\lambda^2}{1 + \beta e^x} - k^2 \right\} u = 0 \quad \dots \quad (2)$$

$$\text{where, } x = \frac{r}{a}, \quad \lambda^2 = \frac{2m}{\hbar^2} V_0 a^2$$

$$\beta = e^{-R/a}, \quad k^2 = \frac{2m}{\hbar^2} (U - E) a^2$$

we now seek solutions of the above differential equation of the form

$$u \sim e^{\mp kx} \cdot F_{\mp}$$

Substituting $z = e^{-x}$, we get,

$$z(z + \beta) \frac{d^2F}{dz^2} + (z + \beta)(1 \pm 2k) \frac{dF}{dz} + \lambda^2 F = 0 \quad \dots \quad (3)$$

For our later calculations the independent variable occurring as the argument of the Tshebysheff's polynomial has to be normalised such that it varies from zero to one; so we make the transformation

$$p = \frac{z - z_1}{z_2 - z_1}.$$

We get from equation (3)

$$(p+\mu)(p+\nu) \frac{d^2F}{dp^2} + (p+\nu)(1 \pm 2k) \frac{dF}{dp} + \lambda^2 F = 0, \quad \dots \quad (4)$$

where

$$\mu = \frac{z_1}{z_2 - z_1} \quad \text{and} \quad \nu = \frac{z_1 + \beta}{z_2 - z_1}.$$

we now replace F by a polynomial of finite degree n and following Lanczos equate the differential equation to a term proportional to Tshebysheff's polynomial of order n . We write

$$D(F) = \tau T_n(p)$$

where

$$D \equiv (p+\mu)(p+\nu) \frac{d^2}{dp^2} + (p+\nu)(1 \pm 2k) \frac{d}{dp} + \lambda^2$$

Let

$$F = \sum_{i=0}^n a_i p^i \quad \text{where } a_0 = 1.$$

$$T_n = \sum_{i=0}^n B_i p^i \quad (\text{for values of } B \text{ vide Lanczos p. 140}).$$

Substituting the above expressions for F and T in the differential equation (4) we get the recursion formulae by comparing the coefficients of the same power of p on both sides of the equation.

$$\tau B_r = a_{r+2}[\mu\nu(r+1)(r+2)] + a_{r+1}[(\mu+\nu)r(r+1) + \delta\nu(r+1)] + a_r[r(r-1) + \delta r + \lambda^2].$$

$$\tau B_n = a_n[\lambda^2 + \delta n + n(n-1)]$$

$$\tau B_{n-1} = a_n[n(n-1)(\mu+\nu) + n\delta\nu] + a_{n-1}[(n-1)(n-2) + (n-1)\delta + \lambda^2].$$

where

$$\delta = \begin{cases} 1+2k \\ 1-2k \end{cases}$$

Therefore the solution of the differential equation is known from the recursion formulae except for an arbitrary constant multiplier. The two values of δ give

two solutions. We write therefore for the solution near the surface of the nucleus ($r < r_1$)

$$u = Ae^{-kx}F_-(p) + Be^{+kx}F_+(p). \quad \dots \quad (5)$$

Now for the outside region ($r > r_1$) only Coulomb potential is effective. We can write for this region,

$$\frac{d^2u}{d\rho^2} + \left(1 - \frac{2\eta}{\rho}\right)u = 0, \quad \dots \quad (6)$$

where

$$\rho = \alpha r = \sqrt{\frac{2mE}{\hbar^2}} \cdot r$$

$$2\eta = \frac{2m}{\hbar^2} \cdot \frac{2(Z-2)e^2}{\alpha}$$

We must seek the appropriate solution of the above equation (6) near the boundary ($r = r_1$) where the inner solution ($r < r_1$) is to be matched with the outer solution ($r > r_1$).

The equation has solutions F_0, G_0 the asymptotic behaviour of which is given by

$$F_0 \sim \sin \theta$$

$$G_0 \sim \cos \theta,$$

as

$$\rho \rightarrow \infty,$$

where $\theta = \rho - \eta \log 2\rho + \sigma$, and $\sigma = \arg \Gamma(i\eta + 1)$.

The combination $G_0 + iF_0$ will satisfy our boundary condition that at infinity the α -particle should behave as a free outgoing particle. Now for different ranges defined by values of ρ and η , different representations of F_0 and G_0 are given. In our region ($\rho < 2\eta$) we take the representation of F_0 and G_0 given by Abramowitz, based on Riccati's method as quoted by C. G. Froberg (1955),

$$F_0 = \frac{1}{2}e^{\varphi(t, \eta)}$$

$$G_0 = e^{\psi(t, \eta)},$$

where

$$t = \frac{\rho}{2\eta}$$

$$Q(t, \eta) = 2\eta g_0 + g_1 + (2\eta)^{-1}g_2 + (2\eta)^{-2}g_3 + \dots$$

$$\psi(t, \eta) = -2\eta g_0 + g_1 - (2\eta)^{-1}g_2 + (2\eta)^2g_3 - \dots$$

From continuity of u and $\frac{du}{dr}$ at the point $r = r_1$, we fix the constants A and B of equation (5). It is found that F_0 and $\frac{dF_0}{dr}$ are negligible in comparison with G_0 and $\frac{dG_0}{dr}$ at $r = r_1$. The values of u at $r = r_2$ where $R < r_2 < r_1$, is next calculated from equation (5) and the penetration factor as defined by Blatt and Weisskopf (1954) is as follows

$$P = \frac{1}{|u(r_2)|^2}$$

To the probability of penetration per second we write

$$\lambda = nP$$

where n is the number of times the α -particle hits the barrier wall. If the α -particle moves with a velocity v within the crater of the nucleus of radius R , then $n = \frac{v}{2R}$, further we take the de Broglie wavelength to be equal to $2R$, we obtain (Max Born, 1951).

$$n = \frac{\hbar}{4mR^2}$$

Now the half-life can be calculated from the expression

$$T = \frac{\log_e^2}{\lambda} = \frac{0.6931}{\lambda}.$$

RESULT AND DISCUSSION

The numerical calculations are made for Polonium (RaC') with $A = 214$, $Z = 84$. The values of parameters are the same as used by Igo and Thaler (1957).

$$R = 1.35A^{1/3} + 1.3 \text{ in Fermi} = 8.32 \times 10^{-13} \text{ cm}$$

$$a = 0.5 \times 10^{-13} \text{ cm}, \quad V_0 = 45 \text{ Mev.}$$

$$m = 6.52 \times 10^{-24} \text{ gm.}$$

$$E = 7.714 \text{ Mev.}$$

We have found the value of $P = .059 \times 10^{-16}$. With the same values of the parameters and applying WKB method Rasmussen (1959) found the value of α -emission width $\delta^2 \approx 15$, from which the penetration factor P comes out to be $\sim 1.17 \times 10^{-16}$. Our value is thus about 20 times smaller than this. It is worth while to

mention here that we have taken $U(r)$ as constant equal to $U(r_1)$ for the inner solutions whereas the actual Coulomb contribution for that part would be somewhat larger. Consequently the value of P is expected to be less than the value we have obtained. With our calculated value of the penetration factor the half-life comes out to be

$$T = 3.201 \times 10^{-4} \text{ sec.}$$

which is about twice the experimental value (Rasmussen, 1959)

$$T = 1.636 \times 10^{-4} \text{ sec.}$$

In view of the uncertainty of the values of the parameters, this agreement may be considered quite satisfactory.

The accuracy of WKB method has often been doubted (Blatt, and Weisskopf, 1954). That is why we have not used it and tried Lanczos' method. The result obtained here gives appreciably better agreement than that given by the WKB method.

The application of this technique to the excited states ($l \neq 0$) is under progress.

ACKNOWLEDGMENTS

Authors are indebted to Prof. D. Basu for suggesting the problem and for helpful discussions.

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GRAVITATIONAL FIELD OF DISTANT ROTATING MASSES

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ABSTRACT. In 1958, Thirring calculated the gravitational field near the centre of a rotating spherical shell. The case can be generalised to that of a rotating mass, where the field away from the mass can be determined. Thirring assumed

$$T^{\mu\nu} = \rho v^\mu v^\nu \quad \dots (i)$$

But it can be shown that, starting from a Galilean field we can build up the case a non-Galilean field by introducing some small term in the metric tensor whose Galilean value is unity. Then by a straight forward process, we calculate $T^{\mu\nu}$ given by

$$T^{\mu\nu} = (p_0 + \rho_{00})v^\mu v^\nu - p_0 g^{\mu\nu} \quad \dots (ii)$$

By the introduction of a rotating mass in the galilean field, which now slightly deviates from its original characteristics is given by the metric tensor

$$\left. \begin{aligned} g_{11}, g_{22}, g_{33} &= -(1+\alpha)(1, r^2, r^2 \sin^2\theta) \\ \text{and } g_{44} &= (1-\alpha) \end{aligned} \right\} \quad \dots (iii)$$

The energy-momentum tensor is then calculated with $\alpha = m(1 - r^2 \sin^2\theta \omega^2)$ and it is shown that this satisfies the conditions of mechanics. It is incidentally shown that there is no necessity of introducing $E^{\mu\nu}$ in the expression for $T^{\mu\nu}$, as recently done by Bass and Pirani (1955). The desired results are obtained without making such assumptions.

1. INTRODUCTION AND DERIVATION OF ENERGY— MOMENTUM TENSOR

In a recent paper, Bass and Pirani (1955) and others (1956) have discussed corrections to Thirring's calculations of the gravitational field near the centre of a rotating spherical shell by introducing an additional term $E^{\mu\nu}$ representing the elastic interaction between particles of the shell, in the expression for energy momentum tensor $T^{\mu\nu} = \rho v^\mu v^\nu$, where the symbols on the right hand side have their usual meanings. The method followed by them is an indirect one in so far as addition of $E^{\mu\nu}$ is made to derive the expression for the energy momentum tensor and then certain assumptions are made as to the behaviour of $E^{\mu\nu}$ with reference to the rotating shell. We can however derive the more general expression for the energy-momentum tensor in the form

$$T^{\mu\nu} = (p_0 + \rho_{00})v^\mu v^\nu - g^{\mu\nu}p_0 \quad \dots (1)$$

wherein we can still assume Galilean values for $g_{\mu\nu}$ or $g^{\mu\nu}$. With this and the modifications introduced in the field by the rotating mass, the whole problem can be worked out in a more straight forward manner as given below.

Accordingly, the flat space-time as envisaged in the general theory of relativity defined by the line-element,

$$ds^2 = -\{(dx_0^1)^2 + (dx_0^2)^2 + (dx_0^3)^2 + (dx_0^4)^2\} \quad \dots \quad (2)$$

or its analogue in spherical polar co-ordinates namely,

$$ds^2 = -(dx'_0)^2 - r^2(dx'_1)^2 - r^2 \sin^2\theta (dx'_2)^2 - (dx'_3)^2 \quad \dots \quad (2')$$

We shall use this in preference to (2) and remove the dashes in the equations that follow. By suitable transformation of dx'_0 the 'negative' sign for ρ_{00} which may otherwise appear can be avoided. Energy-momentum tensor can be expressed as :

$$T_0^{\alpha\beta} = \begin{vmatrix} -p_0 & 0 & 0 & 0 \\ 0 & -p_0 & 0 & 0 \\ 0 & 0 & -p_0 & 0 \\ 0 & 0 & 0 & \rho_{00} \end{vmatrix} \quad \dots \quad (3)$$

where $p_{xx}^0 = -p_0$,

$p_{xy}^0 = 0$ etc. etc. as given in equation 85.1, p. 215 of Tolman's Relativity Thermodynamics and Cosmology (1934). If by the appearance of the rotating spherical mass at great distances from the point of interest, the metric for the gravitational field is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad \dots \quad (4)$$

then the energy-momentum tensor for this case can be written down in accordance with the usual transformation law as

$$T^{\mu\nu} = \frac{\partial x^\mu}{\partial x_0^\alpha} \cdot \frac{\partial x^\nu}{\partial x_0^\beta} \cdot T_0^{\alpha\beta} \quad \dots \quad (5)$$

Considering Eqns. (2), (3) and (5) we have,

$$T^{\mu\nu} = \frac{\partial x^\mu}{\partial x_0^1} \cdot \frac{\partial x^\nu}{\partial x_0^1} \cdot p_0 - \frac{\partial x^\mu}{\partial x_0^2} \cdot \frac{\partial x^\nu}{\partial x_0^2} \cdot p_0 - \frac{\partial x^\mu}{\partial x_0^3} \cdot \frac{\partial x^\nu}{\partial x_0^3} \cdot p_0 + \rho_{00} \frac{\partial x^\mu}{\partial x_0^4} \cdot \frac{\partial x^\nu}{\partial x_0^4} \quad \dots \quad (6)$$

Here, the continuity of the $g_{\mu\nu}$ in the sense of three spaces in space-time has not been considered. (See, for instance a paper "Discontinuities in spherically sym-

metric gravitational fields and shells of radiation" by W. Israel where reference in the subject are given.)

Let R_0 be the radius of the rotating shell thus formed and let Ω be its angular velocity of rotation. It may also be assumed that due to the presence of the rotating masses the field deviates only slightly from that in a flat space-time. This will be satisfied approximately if α is considered to be a small quantity of first order.

$$v_0^1 = \frac{dx_0^1}{ds} = 0; v_0^2 = \frac{dx_0^2}{ds} = 0; \frac{v_0^3}{v_0^4} = \frac{\Omega}{i} \quad \dots \quad (7)$$

Initially, we have and $(v_0^4)^2 + R_0^2 \sin^2 \theta (v_0^3)^2 = -1$

$$\text{where } v_0^3 = \frac{dx_0^3}{ds} \text{ and } v_0^4 = \frac{dx_0^4}{ds}$$

The third and the fourth of Eq. (7) above give

$$\left. \begin{aligned} v_0^3 &= \Omega (1 - R_0^2 \Omega^2 \sin^2 \theta)^{-\frac{1}{2}} \\ v_0^4 &= i(1 - R_0^2 \Omega^2 \sin^2 \theta)^{-\frac{1}{2}} \end{aligned} \right\} \quad \dots \quad (8)$$

The metric tensor is now given by the transformation

$$g^{\mu\nu} = \frac{\partial x^\mu}{\partial x_0^\alpha} \cdot \frac{\partial x^\nu}{\partial x_0^\beta} \cdot g_0^{\alpha\beta}$$

which for $g_0^{\alpha\beta}$ given by the metric (2) reduces to

$$g^{\mu\nu} = - \frac{\partial x^\mu}{\partial x_0^1} \cdot \frac{\partial x^\nu}{\partial x_0^1} - \frac{\partial x^\mu}{\partial x_0^2} \cdot \frac{\partial x^\nu}{\partial x_0^2} - \frac{\partial x^\mu}{\partial x_0^3} \cdot \frac{\partial x^\nu}{\partial x_0^3} - \frac{\partial x^\mu}{\partial x_0^4} \cdot \frac{\partial x^\nu}{\partial x_0^4} \quad \dots \quad (9)$$

We can also write the velocity components as

$$\frac{dx^\mu}{ds} = \frac{\partial x^\mu}{\partial x_0^\alpha} \cdot \frac{dx_0^\alpha}{ds} = \frac{\partial x^\mu}{\partial x_0^\alpha} \cdot v_0^\alpha .$$

substituting for v_0^α from Eq. (7) we get

$$\frac{dx^\mu}{ds} = 2i(1 - R_0^2 \Omega^2 \sin^2 \theta)^{-\frac{1}{2}} \frac{\partial x^\mu}{\partial x_0^4} \quad \dots \quad (10)$$

with the help of Eqns. (6) and (9) the energy-momentum tensor can now be written as

$$T^{\mu\nu} = (p_0 + \rho_{00}) \frac{\partial x^\mu}{\partial x_0^4} \cdot \frac{\partial x^\nu}{\partial x_0^4} + g^{\mu\nu} p_0 \quad \dots (11)$$

The positive sign for the second term on the right hand side is due to the particular choice of the metric tensor as indicated in Eq. (2). Substituting for $\frac{\partial x^\mu}{\partial x_0^4}$ in Eq. (11), we get

$$T^{\mu\nu} = -\frac{1}{4}(p_0 + \rho_{00})(1 - R_0^{-2} \Omega^2 \sin^2 \theta)v^\mu v^\nu + g^{\mu\nu} p_0 \quad \dots (12)$$

It is usual to put $idt = dx_0^4$ and $t' = t$, where t' is the time for the modified metric. We can, therefore, rewrite Eq. (10) as

$$\frac{dx^\mu}{ds} = 2(1 - R_0^{-2} \Omega^2 \sin^2 \theta_0)^{-\frac{1}{2}} \frac{\partial x^\mu}{\partial t} \quad \dots (13)$$

Where θ_0 is a certain given value of θ .

2. THE METRIC

It is assumed that the metric inside the rotating mass is only slightly modified and it does not differ very much from the Galilean field. We may, therefore, write

$$ds^2 = -(1 + \alpha)(dz^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + (1 - \alpha)dt^2 \quad \dots (14)$$

where $\alpha = \alpha(r, \theta, t)$. α is an infinitesimal of the first order and its first and second order variations are also small. Their products are infinitesimals of higher order. In the following derivation these are neglected.

It would appear that the solution may be similar to that obtained from Schwarzschild's exterior line-element where it assumed that at large distances from the central particle the space would have symmetrical and isotropic properties. We now proceed to obtain the solution in this case.

For the metric given by (14) the energy-momentum tensor has the following surviving components to the first order of small quantities :

$$KT^1_1 = \frac{1}{r} \frac{\partial \alpha}{\partial r} + \frac{\partial^2 \alpha}{\partial t^2} \quad \dots (15)$$

$$KT^2_2 = KT^3_3 = -\frac{1}{r} \frac{\partial \alpha}{\partial r} + \frac{\partial^2 \alpha}{\partial t^2} \quad \dots (16)$$

$$KT_4^4 = \frac{\partial^2 \alpha}{\partial r^2} + \frac{3}{r} \frac{\partial \alpha}{\partial r} + \frac{1}{r^2} \left(\frac{\partial^2 \alpha}{\partial \theta^2} + \cot \theta \frac{\partial \alpha}{\partial \theta} \right) \quad \dots \quad (17)$$

$$-KT_2^1 = -r^2 KT_1^2 = \frac{1}{2} \frac{\partial^2 \alpha}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial \alpha}{\partial \theta} \quad \dots \quad (18)$$

$$-KT_4^1 = +KT_1^4 = \frac{1}{2} \frac{\partial^2 \alpha}{\partial r \partial t} \quad \dots \quad (19)$$

$$\text{and} \quad -r^2 KT_4^2 = KT_4^2 = \frac{\partial^2 \alpha}{\partial \theta \partial t} - \frac{1}{2} \cot \theta \frac{\partial \alpha}{\partial t} \quad \dots \quad (20)$$

Only first-order infinitesimals are retained in these expressions as α and its derivatives are of the first order. It is also considered that at the point of interest which is at a large distance from the rotating masses the space-time is symmetrical and conditions of isotropy are also satisfied. We have, therefore, $T_1^1 = T_2^2 = T_3^3$,

which give $\frac{\partial \alpha}{\partial r} = 0$. Also, if $T_2^1 = 0 = T_1^2$, $\frac{\partial \alpha}{\partial \theta} = 0$. These latter three relations give $\alpha = 4\chi^2 + \text{constant}$ $\dots \quad (21)$

so that the metric will be given

$$ds^2 = -(1 + \chi^2)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + (1 - 4\chi^2)dt^2 \quad \dots \quad (22)$$

As χ^2 is a function of t only, the transformed metric is;

$$ds^2 = -\{1 + 4[x(t^*)]^2\}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + dt^{*2} \quad \dots \quad (22')$$

3. ANOTHER FORM OF METRIC

The condition that the pressure should be isotropic gives

$$-KT_{\nu}^{\mu} = \begin{vmatrix} \frac{\partial^2 \alpha}{\partial t^2} & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \alpha}{\partial t^2} & 0 & \frac{\cot^2 \theta}{2r^2} \frac{\partial \alpha}{\partial t} \\ 0 & 0 & \frac{\partial^2 \alpha}{\partial t^2} & 0 \\ 0 & -\frac{\cot \theta}{2} \frac{\partial \alpha}{\partial t} & 0 & 0 \end{vmatrix} \quad \dots \quad (23)$$

where $\alpha = 4\chi^2 + \text{constant}$; $\chi^2 = \frac{1}{1 - R_0^2 \sin^2 \theta_0 \Omega^2}$; $\frac{\partial \alpha}{\partial t} = 8R_0^2 \sin^2 \theta \chi^4 \Omega \dot{\Omega}$; and

$$\frac{\partial^2 \alpha}{\partial t^2} = 8R_0^2 \sin^2 \theta \chi^4 \{\Omega \ddot{\Omega} - 3\dot{\Omega}^2 + 4\Omega^2 \chi^2\}$$

The density ρ_{00} turns out to be zero to a first approximation as would naturally be expected. We may however consider that the condition of isotropy is not strictly satisfied and that $T_2^1 = 0 = T_1^2$, then, a particular solution can be found viz.,

$$\alpha = m(1 - r^2 \sin^2 \theta \cdot \omega^2) \quad \dots \quad (24)$$

where m is an infinitesimal constant and we may take to depend upon time. Obviously ω may be considered as the angular velocity. The energy-momentum tensor can now be written as

$$-KT_{\mu\nu} = \begin{vmatrix} 2m \sin^2 \theta \{ \omega^2 + r^2(\ddot{\omega} + \dot{\omega}^2) \}, & 0 & 0 & 2mr \sin^2 \theta \omega \dot{\omega} \\ 0 & -2mr^2 \sin^2 \theta \{ \omega^2 - r^2(\ddot{\omega} + \dot{\omega}^2) \}, & 0 & 3mr^2 \sin \theta \cos \theta \omega \dot{\omega} \\ 0 & 0, & -2mr^2 \sin^4 \theta \{ \omega^2 - r^2(\ddot{\omega} + \dot{\omega}^2) \}, & 0 \\ 2mr \sin^2 \theta \omega \dot{\omega}, & 3mr^2 \sin \theta \cos \theta \omega \dot{\omega}, & 0, & 2m \{ \omega^2 + r^2 \sin^2 \theta (\ddot{\omega} + \dot{\omega}^2) \} \end{vmatrix} \quad (25)$$

The mechanical relation which must be satisfied by the energy-momentum tensor is given by $\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0$. It will be seen that this gives $\frac{\partial T^{33}}{\partial \phi} = 0$, which is satisfied in the present case. The other relations are :

$$\left. \begin{aligned} \frac{\partial T^{11}}{\partial r} + \frac{\partial T^{14}}{\partial t} &= 0 \\ \frac{\partial T^{22}}{\partial \theta} + \frac{\partial T^{24}}{\partial t} &= 0 \\ \text{and} \quad \frac{\partial T^{44}}{\partial t} + \frac{\partial T^{41}}{\partial r} + \frac{\partial T^{42}}{\partial \theta} &= 0 \end{aligned} \right\} \quad \dots \quad (26)$$

which gives a differential equation for the determination of ω namely,

$$\ddot{\omega} + a\dot{\omega}^2 + b\omega^2 = 0 \quad \dots \quad (27)$$

where a and b are constant coefficients.

R E F E R E N C E S

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DIELECTRIC RELAXATION IN RELATION TO TEMPERATURE. II

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ABSTRACT. In continuation of the previous work, the effect of temperature on the relaxation time τ of three more polar molecules in the non-polar solvent heptane was studied adopting now the Cole-Cole arc plot method for finding τ . The molecules investigated are 2, 3-dichloro nitrobenzene, 2, 5-dichloro nitrobenzene, and 6-chloro 2-nitrotoluene for which the results are newly obtained. The results are examined in the light of Eyring's theory.

In the previous paper (Sobhanadri, 1959) describing the results of experiments on the effect of temperature on dielectric relaxation in six molecules it was shown with reference to Eyring's equation

$$\tau = A/T \cdot e^E \tau/kT = h/kT \cdot e^E \tau/kT$$

(1) that the factor A for a given liquid remains constant and independent of temperature, (2) that its value is considerably higher than that calculated from the equation above, namely h/k , (3) that it is different for different liquids and (4) that the value of E is less than the corresponding value of $E\eta$.

These conclusions were drawn from experimental determinations of τ based on Whiffen and Thompson's (1946) method of observation at a single frequency. However, as pointed out already, it is necessary in the work of this kind to determine τ with as high an accuracy as is possible and perhaps Cole and Cole's (1941) method based on measurements at different frequencies is the most suitable. It is, therefore, thought worthwhile carrying out further investigations using this method. The purpose of this paper is to describe such investigations on the three molecules 2, 5-dichloro nitrobenzene, 2, 3-dichloronitrobenzene, and 6-chloro 2-nitrotoluene making measurements on solutions in heptane at 1.22 cm, 3.26 cm and 4.36 cm.

The experimental arrangement is the same as at 1.22 cm described in the previous paper Sobhanadri (1959). A solution of known concentration is taken in the dielectric cell. The temperature is varied between 0°C and 52°C. In order to avoid errors arising from divergences and variations in the concentration, the experimental determination of x_0 , Δx and ϵ_0 are made with the same prepared solution in rapid succession at the three wavelengths.

The values of ϵ' and ϵ'' at the different wavelengths together with the static value ϵ_0 of the solution (measured at 1 Mc/sec) are presented in Table I for different temperatures for each liquid. The plots for each substance at different temperatures are shown in Figs. 1 to 3.

It is seen that the conclusions referred to in the previous experiments are confirmed by the present results that (1) for a given liquid there is no systematic

TABLE I
(1) 6-chloro 2-nitrotoluene

Temp °C	ϵ_0	1.22 cm		3.26 cm		4.36 cm		$\tau \times 10^{12}$		
		ϵ'	ϵ''	ϵ'	ϵ''	ϵ'	ϵ''	sec	$A \times 10^{11}$	
0	2.029	1.948	0.026	1.975	0.044	1.991	0.043	20.13	10.71	
20	2.018	1.950	0.029	1.980	0.036	1.993	0.025	15.33	11.41	
30	2.007	1.949	0.026	1.979	0.033	1.993	0.033	12.19	10.96	
39	2.001	1.956	0.029	1.980	0.032	1.991	0.029	10.88	10.14	
52	1.993	1.958	0.029	1.979	0.026	1.989	0.024	8.26	11.39	
		Mean		$A = 10.92 \times 10^{-11}$						
				$E_7 = 1.31 \times 10^{-13}$ ergs						

(2) 2, 3-dichloro nitrobenzene

0	2.092	1.949	0.036	1.988	0.071	2.009	0.076	25.60		22.40	
								17.61	22.00		
18	2.078	1.956	0.045	2.008	0.069	2.029	0.067				
30	2.063	1.958	0.046	2.011	0.066	2.030	0.061	15.93	22.33		
40	2.044	1.960	0.048	2.017	0.061	2.033	0.055	13.81	25.80		
52	2.027	1.968	0.046	2.023	0.055	2.036	0.047	11.58	20.30		
		Mean		$A = 22.57 \times 10^{-11}$				$E_7 = 1.33 \times 10^{-13}$ ergs			

(2) 2, 5-dichloro nitrobenzene

0	2.056	1.948	0.030	1.981	0.055	1.998	0.058	23.43		17.13	
								17.61	17.69		
20	2.041	1.940	0.031	1.989	0.051	2.004	0.049				
30	2.036	1.945	0.033	1.984	0.047	1.998	0.044	16.32	18.23		
40	2.022	1.948	0.033	1.984	0.041	1.995	0.036	14.28	17.94		
54	2.014	1.948	0.034	1.984	0.037	1.993	0.032	12.76	18.16		
		Mean		$A = 17.83 \times 10^{-11}$				$E_7 = 1.33 \times 10^{-13}$ ergs			

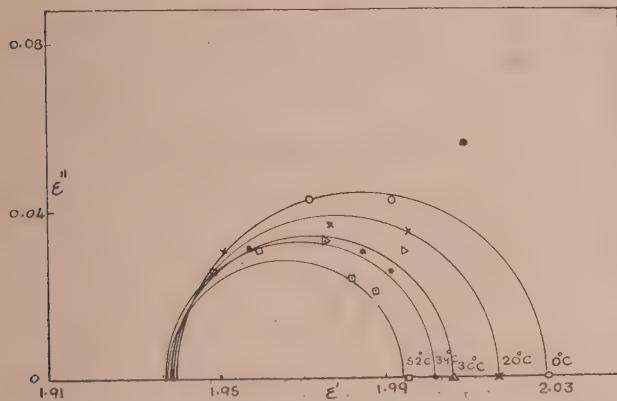


Fig. 1. 6-Chloro-2-nitrotoluene.

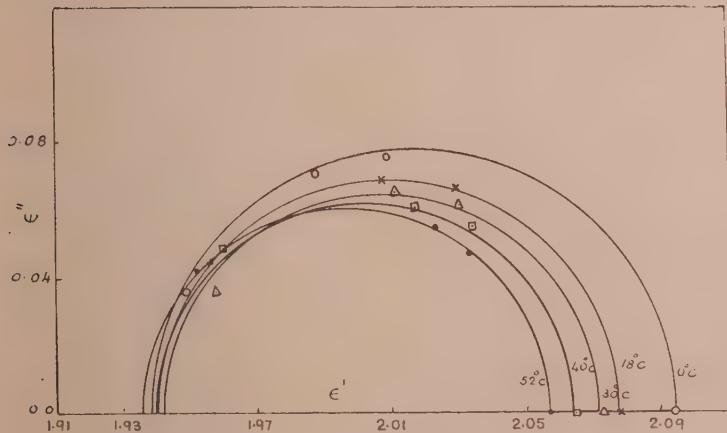


Fig. 2. 2,3-Dichloro-nitrobenzene.

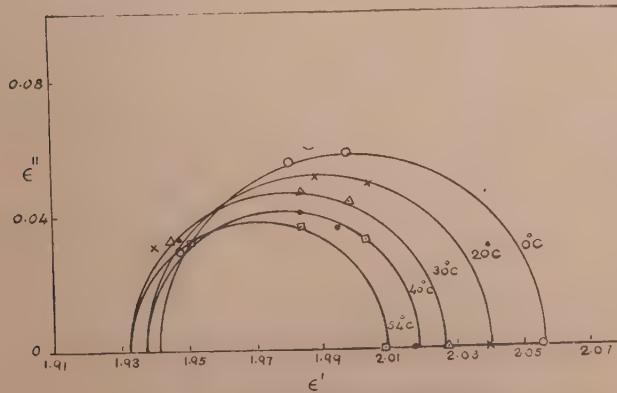


Fig. 3. 2,5-Dichloro-nitrobenzene.

variation in A , its constancy may be assumed, (2) the values are considerably higher than the theoretical value 4.5×10^{-11} and (3) A is different for different liquids.

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FOURTH ORDER SCATTERING MATRIX ELEMENTS OF NUCLEONS WITH A FOURTH ORDER MESON EQUATION

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ABSTRACT. In this paper the important fourth order matrix elements with a fourth order meson equation have been calculated. It is noted that the charge renormalisation that we obtain with this theory is finite and small for a reasonable value of the coupling constant. But the contribution from the meson self-energy diagram remains unaltered and thus contains an infinity. A consistent interpretation of this term as meson mass renormalisation is obtained only if the bare meson has two different rest-masses.

INTRODUCTION

In two previous papers, we have considered a fourth order meson equation proposed by Bhabha (1950) and Thirring (1950). It has been noted that this theory is fairly successful for the explanation of the anomalous magnetic moments of nucleons (Misra and Deo, 1956), but fails to give experimental results for neutronproton scattering (Misra, 1960). However, the principal advantage of this theory is the finiteness of many matrix elements that are divergent in conventional meson theory. For example, Thirring noted that the self-energy of the nucleon in this theory was finite and fairly small for a reasonable value of the coupling constant. These are well illustrated when we consider the fourth order matrix elements for nucleon-nucleon scattering. It is the purpose of the present paper to calculate these matrix elements.

It is noted here that the charge renormalisation that we obtain in this theory from the vertex diagram is finite and small. But, the contribution from the meson self-energy diagram remains unaltered, and thus contains an infinity. This happens since the nucleon propagator, on which this depends, remains the same here. The interpretation of this term as mass renormalisation, however, presents serious difficulty, and this infinite renormalisation has been carried out in the appendix. We find that this mass renormalisation can be consistently done to give rise to unique renormalised mass only if the bare mesons had two different rest-masses. On the other hand, a single mass of the bare meson would give rise to two different renormalised masses. This aspect of the problem would be inter-

esting if the renormalisation terms were finite, which unfortunately is not the case here.

We now proceed to evaluate the fourth order matrix elements. The notation here will be the same as in the previous paper (Misra, 1960). We only note that the nucleon propagator is given as

$$\langle P(\psi(x)\bar{\psi}(y)) \rangle_0 = \frac{i}{(2\pi)^4} \int \frac{i\gamma k + \kappa_0}{(k^2 - \kappa_0^2)} \exp(ik(x-y)) d^4k \quad \dots \quad (1)$$

and the meson propagator, as

$$\langle P(\phi^i(x)\phi^j(y)) \rangle_0 = -\frac{i\kappa^2}{(2\pi)^4} \delta_{ij} \int \frac{\exp(ik(x-y))}{(k^2 + \kappa^2)^2} d^4k \quad \dots \quad (2)$$

FOURTH ORDER S-MATRIX ELEMENTS

Here we have,

$$\langle S_4 \rangle = \frac{f^4}{4!} \int d^4x_1 \dots \int d^4x_4 \langle P\bar{\psi}(x_1)\gamma_5\tau_i\psi(x_1)\phi^i(x_1) \dots \bar{\psi}(x_4)\gamma_5\tau_i\psi(x_4)\phi^i(x_4) \rangle \quad \dots \quad (3)$$

where the expectation value is to be taken between initial and final two-nucleon states with four momenta p_1, p_2 and p_3, p_4 respectively. This gives rise to different Feynman diagrams which we consider separately.

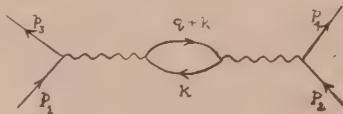


Fig. 1(a)

(a) *Vacuum polarisation :*

Here we obtain

$$\begin{aligned} \langle S_4 \rangle &= \frac{f^4 \kappa^4}{4!} \cdot 4C_2 \cdot 2 \cdot \frac{\kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{\frac{1}{2}}} \delta(p_1 + p_2 - p_3 - p_4) \\ &\times \bar{u}(p_3)\gamma_5\tau_i u(p_1)\bar{u}(p_4)\gamma_5\tau_j u(p_2) \times \int d^4k \{ S_p [\gamma_5\tau_i(i\gamma(q+k) - \kappa_0)\gamma_5\tau_j(i\gamma k - \kappa_0)] \\ &(q^2 + \kappa^2)^{-4} ((q+k)^2 + \kappa_0^2)^{-1} (k^2 + \kappa_0^2)^{-1} \}. \end{aligned}$$

which simplifies to

$$\begin{aligned} S_4^{(a)} &= \frac{4f^4 \kappa^4 \kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{\frac{1}{2}}} \delta(p_1 + p_2 - p_3 - p_4) (q^2 + \kappa^2)^{-4} I \\ &\times \bar{u}(p_3)\gamma_5\tau_i\psi(p_1)\bar{\psi}(p_4)\gamma_5\tau_i\psi(p_2) \quad \dots \quad (4) \end{aligned}$$

where

$$I = \int d^4k \frac{(k^2 + qk + \kappa_0^2)}{(k^2 + 2qk + q^2 + \kappa_0^2)(k^2 + \kappa_0^2)}$$

The above integral is divergent, and we can write, as has been shown in the appendix,

$$I \equiv I(q^2) = A + (q^2 + \kappa^2)B + (q^2 + \kappa^2)^2 I_c(q^2). \quad \dots \quad (6)$$

where A and B are respectively quadratically and logarithmically divergent constants, and $I_c(q^2)$ gives rise to a finite contribution. We shall also show in the appendix that both A and B go as mass renormalisation terms. We note that the same constants also occur in conventional meson theory; but there the interpretation is different, A alone going as mass renormalisation, and B giving rise to coupling constant renormalisation (Schweber, Bethe and Hoffman, 1955).

After renormalisation, the (finite) contribution from this diagram has been evaluated in the appendix as

$$\begin{aligned} (S^{(a)}_{\text{4}})_{\text{rn}} = & \frac{4f^4\kappa_0^2\kappa^4}{(2\pi)^6(p_{10}p_{20}p_{30}p_{40})^{1/2}} \delta(p_1 + p_2 - p_3 - p_4) \\ & \times \bar{u}(p_3)\gamma_5\tau_i u(p_1) \bar{u}(p_4)\gamma_5\tau_i u(p_2)(q^2 + \kappa^2)^{-2} \\ & \times i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz x^2(1-x)^2y^2 \{3(\kappa_0^2 + q^2x(1-x))yz \\ & - \kappa^2x(1-x)(1-yz))^{-1} + 2\kappa_0^2(\kappa_0^2 + q^2x(1-x))yz - \kappa^2x(1-x)(1-yz)^{-1}\} \dots \quad (7) \end{aligned}$$

In the nonrelativistic approximation, neglecting terms of the order of (q^2/κ_0^2) and (κ^2/κ_0^2) as compared to unity, we get

$$\begin{aligned} (S^{(a)}_{\text{4}})_{\text{rn}} = & \frac{2i\pi^2 f^4 \kappa^4}{9\kappa_0^2 (2\pi)^6} \delta(p_1 + p_2 - p_3 - p_4)(q^2 + \kappa^2)^{-2} \times \\ & \times \bar{u}(p_3)\gamma_5\tau_i u(p_1)\bar{u}(p_4)\gamma_5\tau_i u(p_2). \end{aligned}$$

Thus we find that this contribution, which is of the same type as the second order matrix element, is of the order of $(f\kappa/\kappa_0)^2$ times the second order matrix element.

(b) *Vertex diagram :*

We see that the contribution from the vertex diagram (Fig. 1b) is

$$S^{(b)}_4 = \frac{f^4 \kappa^4 \kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{\frac{1}{2}}} \delta(p_1 + p_2 - p_3 - p_4) (q^2 + \kappa^2)^{-2} \int d^4 k \bar{u}(p_3) \gamma_5 \tau_i$$

$$\times (i\gamma(p_3 - k) - \kappa_0) \gamma_5 \tau_j (i\gamma(p_1 - k) - \kappa_0) \gamma_5 \tau_i u(p_1) (k^2 + \kappa^2)^{-2} \\ \times ((k - p_3)^2 + \kappa_0^2)^{-1} ((k - p_1)^2 + \kappa_0^2)^{-1} \bar{u}(p_4) \gamma_5 \tau_j u(p_2),$$

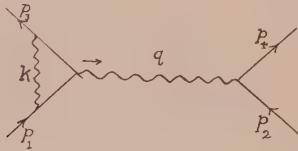


Fig. 1(b)

which simplifies to

$$S^{(b)}_4 = - \frac{f^4 \kappa^4 \kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{\frac{1}{2}}} \delta(p_1 + p_2 - p_3 - p_4) \bar{u}(p_3) \gamma_5 \tau_i u(p_1) \bar{u}(p_4) \gamma_5 \tau_i u(p_2) \\ \times (q^2 + \kappa^2)^{-2} U \dots \quad (8)$$

where

$$U = \int \frac{k^2 d^4 k}{(k^2 - 2kp_1)(k^2 - 2kp_3)(k^2 + \kappa^2)^{-2}} \dots \quad (9)$$

The above integral U is convergent. To evaluate it, we use

$$1/(abc^2) = \int_0^1 dx \int_0^1 dy 6y(1-y) \{(ax + b(1-x))y + c(1-y)\}^{-4},$$

and thus obtain, on integrating for the momentum variable,

$$U = i\pi^2 \int_0^1 dx \int_0^1 dy y(1-y) \frac{y^2(\kappa_0^2 + q^2x(1-x)) + 2\kappa^2(1-y)}{(y^2(\kappa_0^2 + q^2x(1-x)) + \kappa^2(1-y))^2} \dots \quad (10)$$

We can carry out charge renormalisation by writing

$$U = i\pi^2 \int_0^1 dx \int_0^1 dy y(1-y) \{(y^2\kappa_0^2 + \kappa^2(1-y))^{-1} + \kappa^2(1-y)(y^2\kappa_0^2 + \kappa^2(1-y))^{-2} \\ + i\pi^2 \int_0^1 dx \int_0^1 dy y(1-y) \{(y^2(\kappa_0^2 + q^2x(1-x)) + \kappa^2(1-y))^{-1} \\ - (y^2\kappa_0^2 + \kappa^2(1-y))^{-1} - \kappa^2(1-y)(y^2\kappa_0^2 + \kappa^2(1-y))^{-2} \\ + \kappa^2(1-y)(y^2(\kappa_0^2 + q^2x(1-x)) + \kappa^2(1-y))^{-2}\}$$

$$\begin{aligned}
&= i\pi^2 \int_0^1 dx \int_0^1 dy y(1-y) \frac{y^2 \kappa_0^2 + 2\kappa^2(1-y)}{(y^2 \kappa_0^2 + \kappa^2(1-y))^2} - i\pi^2 q^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz y(1-y) \\
&\quad \times x(1-x) \{ (y^2 \kappa_0^2 + q^2 x(1-x) y^2 z + \kappa^2(1-y))^{-2} \\
&\quad + 2\kappa^2(1-y) (y^2 \kappa_0^2 + q^2 x(1-x) y^2 z + \kappa^2(1-y))^{-3} \} . \quad \dots \quad (11)
\end{aligned}$$

In deducing the above result we have utilised the equation

$$\frac{1}{\alpha^n} - \frac{1}{\beta^n} = \int_0^1 \frac{n(\beta-\alpha)}{\{\alpha z + \beta(1-z)\}^{n+1}} dz . \quad \dots \quad (12)$$

The first term on the right hand side of Eq. (11) may be regarded as the charge renormalisation term to be included in the second order contribution. This is because we assume that when there is no change in momentum, the contribution from the vertex diagram should vanish. The second term of the same equation gives us the physically important contribution from this process. For non-relativistic regions, neglecting q^2 as compared to κ_0^2 and making the substitution $\lambda = \kappa^2/\kappa_0^2$, the second term of Eq. (11) is seen to be

$$\begin{aligned}
&\simeq -i\pi^2 q^2 \frac{1}{6\kappa_0^4} \int_0^1 \left\{ \frac{y(1-y)dy}{(y^2 + \lambda(1-y))^2} + \frac{2\lambda y(1-y)^2 dy}{(y^2 + \bar{\lambda}(1-y))^3} \right\} \\
&\simeq -\frac{i\pi^2 q^2}{6\kappa_0^4} \left[\frac{1}{\lambda} - \frac{3}{8\lambda^{\frac{1}{2}}} \cos^{-1}(\frac{1}{2}\lambda^{\frac{1}{2}}) \right] .
\end{aligned}$$

Thus we have, $U = O\left(\frac{q^2}{\kappa_0^2 \kappa^2}\right)$, and hence, substituting in Eq. (8) and comparing with the second order contribution, the physical part of the contribution from the vertex diagram is found to be of the order of $f^2 q^2/\kappa_0^2$ times the second order contribution.

We shall now see the order of magnitude of the renormalisation term. This term, on the right hand side of Eq. (11), is

$$\begin{aligned}
&\frac{i\pi^2}{\kappa_0^2} \left[\int_0^1 \frac{y^3(1-y)}{(y^2 + \lambda(1-y))^2} dy + 2\lambda \int_0^1 \frac{y(1-y)^2}{(y^2 + \lambda(1-y))^2} dy \right] \\
&\simeq \frac{i\pi^2}{\kappa_0^2} [0.8 + 0.7] = \frac{i\pi^2}{\kappa_0^2} \times 1.5 .
\end{aligned}$$

Thus, by Eqns. (8) and (11), the part of $S^{(b)}_4$ that goes as renormalisation term is

$$\begin{aligned}
&-\frac{f^4 \kappa^4 \kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{\frac{1}{2}}} \frac{i\pi^2}{\kappa_0^2} \times 1.5 \times \delta(p_1 + p_2 - p_3 - p_4) (q^2 + \kappa^2)^{-2} \\
&\times \bar{u}(p_3) \gamma_5 \tau_i u(p_1) \bar{u}(p_4) \gamma_5 \tau_i u(p_2) .
\end{aligned}$$

Hence, on comparing with the second order contribution, we obtain,

$$\begin{aligned} f^2(\text{renormalised}) &= f^2 \left[1 + \frac{f^2 \kappa^2}{16\pi^2 \kappa_0^2} \times 1.5 \right] \\ &= f^2 [1 + f^2 \times 2.14 \times 10^{-4}]. \end{aligned}$$

This gives us that the renormalisation of the coupling constant is small for any reasonable value of the bare coupling constant, which in this case must be finite. We note that here the bare coupling constant is not an abstraction to be discarded later on, but is a meaningful quantity that can be determined to any degree of accuracy in terms of the physical or renormalised coupling constant.

(c) *Nucleon self-energy* :

The contribution from the nucleon self-energy graph (Fig. 1c) is to be treated just the same way as for ordinary mesons theory, and is to be taken along with the contribution from the diagram Fig. 1c'. We may note that the contribution from both the diagrams gives rise to an ambiguous expression and to get meaningful results we may assume some form of periodic damping explicitly. For example, we may take our interaction Hamiltonian as (Schweber, Bethe and de Hoffman, 1955, p. 286)

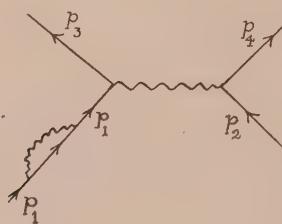


Fig. 1(c)

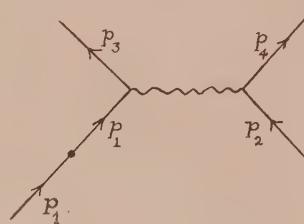


Fig. 1(c')

$$H_i(x) = i f g(t) \bar{\psi}(x) \gamma_5 \tau_i \psi(x) \phi^i(x) - \delta \kappa_0 (g(t))^2 \bar{\psi}(x) \psi(x) \quad (13)$$

where

$$g(t) = \int_{-\infty}^{\infty} G(\Gamma_0) e^{-i \Gamma_0 t} d\Gamma_0 = \int_{-\infty}^{\infty} G(\Gamma_0) e^{+i \Gamma_0 x} d\Gamma_0 \quad \dots \quad (14)$$

with $\Gamma = (\Gamma_0, 0, 0, 0)$ and

$$g(0) = \int_{-\infty}^{\infty} G(\Gamma_0) d\Gamma_0 = 1.$$

As an example of a function of such a type, we can take

$$g(t) = (T/t) \sin(t/T) \quad \dots \quad (15)$$

such that

$$G(\Gamma_0) = \begin{cases} T/2 & \text{when } -1/T \leq \Gamma_0 \leq 1/T \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, in the limit T approaches infinity, $G(\Gamma_0)$ approaches a δ -function, and for any finite time, $g(t)$ approaches unity.

Let us now consider the contribution from the self-energy diagram in the general case, with p not the momentum of an external line, given as

$$\begin{aligned} \Sigma(p) &= \frac{3if^2\kappa^2}{(2\pi)^4} \int \frac{-i\gamma(p-k)}{((p-k)^2 + \kappa_0^2)(k^2 + \kappa^2)^2} d^4k \\ &= A + B(i\gamma p + \kappa_0) + (i\gamma p + \kappa_0)^2 \Sigma_e(p), \text{ say.} \end{aligned} \quad \dots \quad (16)$$

Now, when we recalculate the contributions from the Figs. 1(c) and 1(c') with the Hamiltonian given by Eq. (13), by interpretation of the term for mass renormalisation, the constant A in Eq. (16) will exactly cancel with the term before going the limit $T \rightarrow \infty$. Also, the constant B in this equation, which at the outset is ambiguous, becomes now well-defined and gives rise to the wavefunction renormalisation (also vide Juch and Rohrlich, 1955, p. 185). Again, since in this case we have an external line, the contribution with Σ_e will not be there.

Thus, with such diagrams, the contribution goes only to a mass renormalisation (which is finite and has been calculated by Thirring (1950)) and to a wavefunction renormalisation, which also is finite and can be calculated in the standard way), and there is no contribution to the scattering process.

(d) *Successive and crossed exchange of two mesons :*

As in the previous cases, the contribution from the Feynman diagram Fig. 1(d) for the case of successive exchange of two mesons becomes, on using the equations with p_1 and p_2 as the four-momenta of free particles,

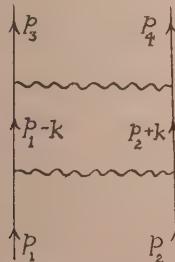


Fig. 1(d)

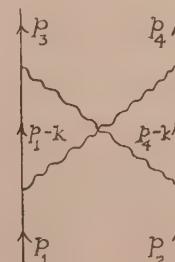


Fig. 1(e)

$$S_4^{(2)} = \frac{f^4 \kappa^4 \kappa_0^2}{(2\pi)^4 (p_{10} p_{20} p_{30} p_{40})^2} \delta(p_1 + p_2 - p_3 - p_4) V_{\mu\nu}$$

$$\times \bar{u}(p_3) \tau_i \tau_j \gamma_\mu^{(1)} u(p_1) \bar{u}(p_4) \tau_i \tau_j \gamma_\mu^{(2)} u(p_2), \quad \dots \quad (17)$$

where

$$V_{\mu\nu} = \int d^4k \frac{k_\mu k_\nu}{(k^2 + 2kp_2)(k^2 - 2kp_1)((k-q)^2 + \kappa^2)^2(k^2 + \kappa^2)^2} \quad \dots \quad (18)$$

Using representation with subsidiary variables and carrying out integration over the momentum variable, we obtain, on simplification,

$$\begin{aligned} V_{\mu\nu} &= 120 \int_0^1 dx \int_0^1 dy \int_0^1 dz z(1-z) x(1-x)^3 \int d^4k k_\mu k_\nu [k^2 - 2k\{p_1x(1-y) \\ &\quad - p_2xy + qz(1-x)\} + q^2z(1-x) + \kappa^2(1-x)]^{-6} \\ &= i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz x(1-x)^3 z(1-z) \left[\frac{6Q_\mu Q_\nu}{D^4} + \frac{g_{\mu\nu}}{D^3} \right], \end{aligned} \quad \dots \quad (19)$$

where

$$Q = p_1x(1-y) - p_2xy + qz(1-x) \quad \dots \quad (20)$$

and

$$D = \kappa_0^2 x^2(1-2y)^2 + \kappa^2(1-x) + q^2z(1-z)(1-x)^2 - p^2 x^2 y(1-y). \quad \dots \quad (21)$$

Substituting the value of Q from equation (20), we see that in equation (17),

$$\gamma_\mu^{(1)} \gamma_\nu^{(2)} Q_\mu Q_\nu \simeq -\kappa_0^2 x^2(1-2y)^2$$

in the nonrelativistic approximation. Also, in this approximation,

$$\gamma_\mu^{(1)} \gamma_\nu^{(2)} g_{\mu\nu} \simeq 1.$$

Hence we have in Eqns. (17) and (19),

$$\gamma_\mu^{(1)} \gamma_\nu^{(2)} V_{\mu\nu} = i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz x(1-x)^3 z(1-z) \times \left\{ \frac{-6\kappa_0^2 x^2(1-2y)^2}{D^4} + \frac{1}{D^3} \right\} \quad (22)$$

where D is given by eqn. (21).

The evaluation of the above integral even in the non-relativistic approximation requires special care, since the denominator vanishes for certain values of the auxiliary variables x and y . The singularity here is to be dealt with by adding a small negative imaginary part to the mass of the meson. The integral to be evaluated is

$$I = \int_0^1 dx \int_0^1 dy \int_0^1 dz x(1-x)^3 z(1-z) \left\{ -\frac{6\kappa_0^2 x^2(1-2y)^2}{D^4} + \frac{1}{D^3} \right\},$$

where, for Eq. (21), we can take

$$D \simeq \kappa_0^2 x^2(1-2y)^2 + \kappa^2(1-x) + q^2z(1-z)(1-x)^2 - \frac{1}{4} p^2 x^2 \quad \dots \quad (23)$$

We now note that

$$I = \frac{\partial}{\partial q^2} \frac{\partial}{\partial \kappa^2} I_0, \quad \dots \quad (24)$$

where

$$I_0 = \int_0^1 dz \int_0^1 dx \int_0^1 dy \left\{ -\frac{\kappa_0^2 x^2 (1-2y)^2}{D^2} + \frac{1}{2} D^{-1} \right\} \quad \dots \quad (24')$$

We now put for brevity

$$\phi(x, z) \equiv \kappa^2(1-x) + q^2z(1-z)(1-x)^2 - \frac{1}{4}p^2x^2 \quad \dots \quad (25)$$

and put $s = \kappa_0 x(1-2y)$ as the integration variable instead of y . Then we get,

$$I_0 = \int_0^1 dz \int_0^1 dx \int_0^{\kappa_0 x} ds \left\{ -s^2/(s^2 + \phi(x, z))^{-2} + \frac{1}{2}(s^2 + \phi(x, z))^{-1} \right\} \quad \dots \quad (26)$$

Since p^2 and q^2 are positive definite constants, (they are positive definite in the centre of mass system), we have, in Eq. (25), $\phi(x, z)$ is a monotonically diminishing function of x for any given value of z , and is positive when $x = 0$ and negative when $x = 1$. Hence $\phi(x, z)$ vanishes for one value $\xi = \xi(z)$ of x , with $0 < \xi < 1$, and we have

and $\phi(x, z) > 0$ when $0 < x < \xi$

$\phi(x, z) < 0$ when $\xi < x < 1$.

In the second case above, the singularity for the s -integration is to be treated by the addition of a small negative imaginary part. Separating the region of integration of x into $(0, \xi)$ and $(\xi, 1)$ and performing the s -integration for these regions separately, we obtain, after some lengthy but straightforward calculations,

$$I_0 = \frac{1}{2} \int_0^1 dz \int_0^1 dx \frac{x}{[\kappa_0^2 x^2 + \kappa^2(1-x) + q^2z(1-z)(1-x)^2 - \frac{1}{4}p^2x^2]} \quad \dots \quad (27)$$

Thus, by Eqns. (24) and (24'),

$$I \simeq \int_0^1 dz \int_0^1 dx \frac{x(1-x)^3 z(1-z)}{[\kappa_0^2 x^2 + \kappa^2(1-x) + q^2z(1-z)(1-x)^2]^3}, \quad \dots \quad (28)$$

where we have again neglected $\frac{1}{4}p^2$ as compared to κ_0^2 .

Now, in Eq. (28), we note that when x in $(1-x)$ of the denominator becomes significant, the term $\kappa_0^2 x^2$ becomes large compared with the other terms, whereas, for small values of x the terms κ^2 and q^2 predominate in the denominator. Hence in the denominator, we can write 1 instead of $(1-x)$ for the factors of κ^2 and q^2 . Again, we can omit x in $(1-x)^3$ in the numerator, since, when x is comparable with unity, the κ_0 in the denominator makes the contribution anyhow small even without this factor, and thus the change thus introduced is negligible. Thus we can write

$$I \simeq \int_0^1 dz \int_0^1 dx \frac{x(1-z)z}{[\kappa_0^2 x^2 + \kappa^2 + q^2 z(1-z)]^3}. \quad \dots \quad (29)$$

The above heuristic argument, however, does not give us the degree of error in taking the value (29) instead of (28). We can estimate this by taking the difference of the right hand sides of Eqns. (28) and (29). Using Eq. (12) we can show that this difference is of the order of

$$\int_0^1 \frac{x^2 dx}{[\kappa_0^2 x^2 - Ax + B]^3} + O(\kappa^2) \int_0^1 \frac{x^2 dx}{[\kappa_0^2 x^2 - Ax + B]^4}, \quad \dots \quad (30)$$

where the quantities A and B do not depend on x and are of the order of κ^2 with $B > A$. Starting with the result

$$J \equiv \int_0^1 \frac{dx}{[\kappa_0^2 x^2 - Ax + B]} = \frac{2}{\sqrt{4\kappa_0^2 B - A^2}} \tan^{-1} \left(\frac{\sqrt{4\kappa_0^2 B - A^2}}{2B - A} \right),$$

we get, by differentiating,

$$\int_0^1 \frac{x^2 dx}{[\kappa_0^2 x^2 - Ax + B]^3} \simeq \frac{1}{16\kappa_0^3 B^{3/2}} \tan^{-1} \left(\frac{2\kappa_0 \sqrt{B}}{2B - A} \right) \quad \dots \quad (31a)$$

and

$$\int_0^1 \frac{x^2 dx}{[\kappa_0^2 x^2 - Ax + B]^4} \simeq \frac{1}{32\kappa_0^2 B^{5/2}} \tan^{-1} \left(\frac{2\kappa_0^2 \sqrt{B}}{2B - A} \right) \quad \dots \quad (31b)$$

Thus we find the difference in the approximations (28) and (29) of the integral I as $O\left(\frac{1}{\kappa_0^3 \kappa^3}\right)$. On the other hand we find that the integral (29) is $O(1/\kappa_0^2 \kappa^4)$, such that we have neglected a term of the order of (κ/κ_0) higher than the leading

term when we have written down Eq. (29). The small values of the coefficients of the leading terms in Eq. (31) further justify this approximation.

Now performing the x -integration in Eq. (29) and retaining the main contribution, we get,

$$I \simeq \frac{1}{4\kappa_0^2} \int_0^1 \frac{z(1-z)dz}{[\kappa^2 + q^2 z(1-z)]^2}, \quad \dots \quad (32)$$

which on direct evaluation gives us

$$1 \simeq \frac{1}{4\kappa_0^2} \left\{ \frac{2\kappa^2 + q^2}{4(q^2)^{3/2}(\kappa^2 + \frac{1}{4}q^2)^{3/2}} \log \left(\frac{\sqrt{\kappa^2 + \frac{1}{4}q^2} + \frac{1}{2}(q^2)^{1/2}}{\sqrt{\kappa^2 + \frac{1}{4}q^2} - \frac{1}{2}(q^2)^{1/2}} \right) - \frac{1}{2q^2(\kappa^2 + \frac{1}{4}q^2)} \right\}, \quad \dots \quad (33)$$

such that by Eqns. (17) and (29),

$$S_4^{(d)} = \frac{i\pi^2 f^4 \kappa^4 \kappa_0^2}{(2\pi)^6 (p_{10} p_{20} p_{30} p_{40})^{1/2}} \delta(p_1 + p_2 - p_3 - p_4) 1 \times \bar{u}(p_3) \tau_i \tau_j u(p_1) \bar{u}(p_4) \tau_i \tau_j u(p_2), \quad \dots \quad (34)$$

For crossed exchange of two mesons, we have, from Fig. 1(e),

$$S_4^{(e)} = -f^4 \kappa^4 (2\pi)^{-6} \kappa_0^2 (p_{10} p_{20} p_{30} p_{40})^{-\frac{1}{2}} \delta(p_1 + p_2 - p_3 - p_4) V'_{\mu\nu} \times \bar{u}(p_3) \tau_i \tau_j \gamma_\mu^{(1)} u(p_1) \bar{u}(p_4) \tau_j \tau_i \gamma_\nu^{(2)} u(p_2), \quad \dots \quad (35)$$

where

$$V'_{\mu\nu} = \int d^4k \frac{k_\mu k_\nu}{(k^2 - 2kp_4)(k^2 - 2kp_1)(k^2 - 2kq + q^2 + \kappa^2)^2 (k^2 + \kappa^2)^2} \quad \dots \quad (36)$$

Proceeding as in the case of successive exchange of two mesons, we obtain,

$$V'_{\mu\nu} = i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \ x(1-x)^3 z(1-z) \left\{ \frac{6Q'_\mu Q'_\nu}{D'^4} + \frac{g_{\mu\nu}}{D'^3} \right\} \quad \dots \quad (37)$$

where

$$Q'_\mu = p_{1\mu} x(1-y) + p_{4\mu} xy + q_\mu z(1-x) \quad \dots \quad (38)$$

and

$$D' = \kappa_0^2 x^2 + \kappa^2(1-x) + p^2 x^2 y(1-y) + q^2 z(1-z)(1-x)^2 + q^2 x^2 y(1-y) \simeq \kappa_0^2 x^2 + \kappa^2(1-x) + q^2 z(1-z)(1-x)^2. \quad \dots \quad (39)$$

In the further nonrelativistic approximation, we have, in Eq. (35),

$$\begin{aligned} \gamma_\mu^{(1)} \gamma_\nu^{(2)} V_{\mu\nu} &= i\pi^2 \int_0^1 dx \int_0^1 dz \quad x(1-x)^3 z(1-z) \left\{ -\frac{6\kappa_0^2 x^2}{D'^4} + \frac{1}{D'^8} \right\} \\ &= i\pi^2 I' \quad \text{say.} \quad \dots \quad (40) \end{aligned}$$

We note that D' is positive definite and hence the integration above does not have the complications of the previous case.

For the evaluation of the integral (40), we make the same approximation as was made in the previous case in going from Eqn. (28) to (29). We have seen that this is strictly valid upto an order $(\kappa \kappa_0)$. With this approximation we have, on carrying out the x -integration,

$$I' = -\frac{1}{4\kappa_0^2} \int_0^1 \frac{z(1-z)dz}{[\kappa^2 - q^2 z(1-z)]^2}. \quad \dots \quad (41)$$

A comparison of the above Eq. with Eq. (32) shows that

$$I' = -I.$$

Hence, by Eqns. (34), (35) and (49) and (41), we obtain, with I given by Eq. (33).

$$S_4^{(d)} + S_4^{(e)} \simeq 6i\pi^2 f^4 \kappa^4 (2\pi)^{-6} \delta(p_1 + p_2 - p_3 - p_4) \bar{I} u(p_3) u(p_1) \bar{u}(p_4) u(p_2).$$

A comparison with the other terms of the fourth order also shows us that the major contribution in the fourth order arises from the successive and crossed exchange of two mesons. This can also be seen from physical reasons. For γ_μ coupling, a transition from particle to antiparticle states will give us the maximum contribution, which is possible for the above type of diagrams.

APPENDIX

Meson mass renormalisation : In order to interpret certain terms arising from Eq. (4) as mass renormalisation, we have to first write the integral (5) occurring there in the form (6). We notice that this integral, written as

$$I = \int_0^1 dx \int d^4k \frac{k^2 + qk + \kappa_0^2}{(k^2 + 2kqx - q^2 x + \kappa_0^2)^2} \quad \dots \quad (A1)$$

is quadratically divergent, and thus requires care even in the change of momentum variables to be integrated. Using well-known methods (Jauch and Rohrlich 1955, also Eq. (12)), we obtain, for a shift of the origin,

$$I = \int_0^1 dx \int d^4k \frac{k^2 - q^2 x(1-x) + \kappa_0^2}{(k^2 + q^2 x(1-x) - \kappa_0^2)^2} - \frac{i\pi^2}{4} q^2 \quad \dots \quad (A2)$$

In order to write the integral I of (A2) in the form (6), we repeatedly make use of the identity (Eq. (12))

$$\frac{1}{\alpha^n} - \frac{1}{\beta^n} = \int_0^1 \frac{n(\beta-\alpha)}{(\alpha z + \beta(1-z))^{n+1}} dz$$

This gives us, after some lengthy calculation and comparing with Eq. (6), the values of the constants A and B as

$$A = \int_0^1 dx \int d^4 k \frac{k^2 + \kappa^2 x(1-x) + \kappa_0^2}{(k^2 - \kappa^2 x(1-x) - \kappa_0^2)^2} + \frac{i\pi^2}{4} \kappa^2 \quad \dots \quad (A3)$$

and

$$B = \int_0^1 dx \int d^4 k \left\{ \frac{-4x(1-x)(k^2 + \kappa_0^2)}{(k^2 - \kappa^2 x(1-x) - \kappa_0^2)^3} + \frac{x(1-x)}{(k^2 - \kappa^2 x(1-x) - \kappa_0^2)^2} \right\} - \frac{i\pi^2}{4} \quad \dots \quad (A4)$$

Also, after momentum integration, the finite integral $I_e(q^2)$ is given as

$$I_e(q^2) = i\pi^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz x^2(1-x)^2 y \sqrt{3(\kappa_0^2 - q^2 x(1-x)yz - \kappa^2 x(1-x)(1-yz))^{-1}} \\ - 2\kappa_0^2(\kappa_0^2 - q^2 x(1-x)yz - \kappa^2 x(1-x)(1-yz))^{-2}. \quad \dots \quad (A5)$$

We shall now see how the divergent constants A and B may be interpreted as mass renormalisation terms. For this purpose, we write the meson field Lagrangian and the interaction Lagrangian in the Heisenberg representation as

$$\underline{L}_m + \underline{L}_i \\ = -\frac{1}{2} [(\square \underline{\phi}^i)(\square \underline{\phi}^i) + (\underline{\phi}_1^2 + \delta \underline{\phi}_1^2 + \underline{\phi}_2^2 + \delta \underline{\phi}_2^2)(\partial_\mu \underline{\phi}_1^i)(\partial_\mu \underline{\phi}_1^i) \\ + (\kappa_1^2 + \delta \kappa_1^2)(\kappa_2^2 + \delta \kappa_2^2)\underline{\phi}_1^i \underline{\phi}_2^i] - if \bar{\psi} \gamma_5 \tau_3 \underline{\psi} \underline{\phi}^i \\ + \frac{1}{2\kappa^2} [(\delta \underline{\phi}_1^2 + \delta \underline{\phi}_2^2)(\partial_\mu \underline{\phi}_1^i)(\partial_\mu \underline{\phi}_2^i) - (\kappa_1^2 \delta \kappa_2^2 - \kappa_2^2 \delta \kappa_1^2 - \delta \kappa_1^2 \delta \kappa_2^2)\underline{\phi}_1^i \underline{\phi}_2^i], \quad \dots \quad (A6)$$

where the line below the operators means the corresponding quantities in Heisenberg representation. In the above, κ_1 and κ_2 are unrenormalised masses and $\delta \kappa_1^2$ and $\delta \kappa_2^2$ are the renormalisation terms, such that the observed mass is given the equations

$$\kappa_1^2 + \delta \kappa_1^2 + \kappa_2^2 + \delta \kappa_2^2 = 2\kappa^2, \\ (\kappa_1^2 - \delta \kappa_1^2)(\kappa_2^2 - \delta \kappa_2^2) = \kappa^4. \quad \dots \quad (A7)$$

We now have to set up the interaction representation and to do this, we proceed as in Umezawa (1956).

Using Eq. (A7), the meson field equation becomes

$$(\square - \kappa^2)^2 \underline{\phi}^i(x) = -if \kappa^2 \bar{\psi}(x) \gamma_5 \tau_i \underline{\psi}(x) + (\kappa_1^2 \delta \kappa_2^2 + \kappa_2^2 \delta + \kappa_1^2 \delta \kappa_2^2) \underline{\phi}^i(x) - (\delta \kappa_1^2 + \delta \kappa_2^2) \square \underline{\phi}^i(x) \quad \dots \quad (A8)$$

We now solve the above equation with the retarded Green's function $G_R(x - x')$ satisfying the equation

$$(\square - \kappa^2)^2 G_R(x - x') = -\delta_4(x - x')$$

and $G_R(x - x') = 0$ when $x_0 < x'_0$ and the function $G(x) = G_R(x) - G_R(-x)$. Then, a partial integration with the assumption that the interaction (and hence the renormalisation terms) vanish at infinite past sufficiently rapidly, gives us,

$$\begin{aligned} \underline{\phi}^i(x) = \phi^i(x) + \frac{1}{2} \int_{-\infty}^{\infty} d^4 x' (1 + \epsilon(x - x')) \{ G(x - x') [if \kappa^2 \bar{\psi}(x') \gamma_5 \tau_i \underline{\psi}(x') \\ - (\kappa_1^2 \delta \kappa_2^2 + \kappa_2^2 \delta \kappa_1^2 + \delta \kappa_1^2 \delta \kappa_2^2) \phi_i(x')] - (\delta \kappa_1^2 + \delta \kappa_2^2) (\partial'_{\mu} G(x - x')) (\partial'_{\mu} \phi^i(x')) \} \quad (A9) \end{aligned}$$

where $\phi^i(x)$ is a field operator satisfying the equation

$$(\square - \kappa^2)^2 \phi^i(x) = 0 \quad \dots \quad (A10)$$

and $\epsilon(x - x') = 1$ or -1 according as x' is earlier or later than x . The words ‘earlier’ and ‘later’ refer to space-like surfaces on which $\epsilon(x - x')$ has a discontinuity.

Now, let the interaction representation state-vector $\underline{\psi}(\sigma)$ be given in terms of the Heisenberg state-vector Φ as $\Phi(\sigma) = S(\sigma)\Phi$, $S(-\infty) = 1$. Then we have,

$$i \frac{\delta S(\sigma)}{\delta \sigma(x)} = H(x, \sigma) S(\sigma) \quad \dots \quad (A11)$$

and the integrability condition

$$\frac{\delta H(x', \sigma)}{\delta \sigma(x)} - \frac{\delta H(x, \sigma)}{\delta \sigma(x')} = i[H(x', \sigma), H(x, \sigma)] \quad \dots \quad (A12)$$

where x and x' are any two points lying on the surface σ .

We now define auxiliary field operators $\phi^i(x, \sigma)$ given by

$$\begin{aligned} \phi^i(x, \sigma) = \phi^i(x) + \int_{-\infty}^{\sigma} \{ G(x - x') [if \kappa^2 \bar{\psi}(x') \gamma_5 \tau_i \underline{\psi}(x') \\ - (\kappa_1^2 \delta \kappa_2^2 + \kappa_2^2 \delta \kappa_1^2 + \kappa_1^2 \kappa_2^2) \phi_i(x')] - (\delta \kappa_1^2 + \delta \kappa_2^2) (\partial'_{\mu} G(x - x')) (\partial'_{\mu} \phi^i(x')) \} d^4 x' \quad (A13) \end{aligned}$$

and proceed exactly as in Umezawa (1956). Then we obtain that the interaction Hamiltonian is given by the equation

$$[\phi(x), H(x', \sigma)] = iG(x-x')S(\sigma)[if \kappa^2\bar{\psi}(x')\gamma_5\tau_i\psi(x') \\ -(\kappa_1^2\delta\kappa_2^2+\kappa_2^2\delta\kappa_1^2+\delta\kappa_1^2\delta\kappa_2^2)\phi^i(x')]S^{-1}(\sigma) \\ -S(\sigma)(\delta_1^2+\delta_2^2)(\partial'_\mu G(x-x'))(\partial'_\mu\phi^i(x'))S^{-1}(\sigma), \dots \quad (A14)$$

where clearly x' lies on σ . Using this, and that $G(x-x')$ and its first order space-time derivatives vanish for space-like separation of the points, Eq. (A14) may be seen to reduce to

$$[\phi^i(x), H(x', \sigma)] = i\kappa^2G(x-x')if \bar{\phi}(x')\gamma_5\tau_i\phi(x') \\ -(\kappa_1^2\delta\kappa_2^2+\kappa_2^2\delta\kappa_1^2+\delta\kappa_1^2\delta\kappa_2^2)iG(x-x')\phi^i(x') \\ -(\delta\kappa_1^2+\delta\kappa_2^2)i(\partial'_\mu G(x-x'))(\partial'_\mu\phi^i(x')) \dots \quad (A15)$$

Since the field operators in Eq. (A15) satisfy free field commutation relations

$$[\phi^i(x), \phi^j(x')] = i\kappa^2\delta_{ij}G(x-x'),$$

we obtain

$$H(x', \sigma) = if\bar{\psi}(x')\gamma_5\tau_i\psi(x')\phi^i(x) - \frac{1}{2}\kappa^2(\kappa_1^2\delta\kappa_2^2+\kappa_2^2\delta\kappa_1^2+\delta\kappa_1^2\delta\kappa_2^2)\phi^i(x')\phi^i(x') \\ -\frac{1}{2}\kappa^2(\delta\kappa_1^2+\delta\kappa_2^2)(\partial'_\mu\phi^i(x'))(\partial'_\mu\phi^i(x')) \dots \quad (A16)$$

We note that the presence of derivatives of field operations in the self-coupling of the meson fields *does not* introduce terms depending on the normal to the surface.

The above interaction Hamiltonian clearly satisfies the integrability condition (A12).

The presence of second and third terms in $H(x, \sigma)$ of Eq. (A16) gives rise to vertices with only two meson lines. Consideration of the corresponding Feynman diagram (Fig. 1f') gives us, with



Fig. 1(f')

$$A_1 = \frac{1}{2}\kappa^2(\kappa_1^2\delta\kappa_2^2+\kappa_2^2\delta\kappa_1^2+\delta\kappa_1^2\delta\kappa_2^2) \\ B_1 = \frac{1}{2}\kappa^2(\delta\kappa_1^2+\delta\kappa_2^2) \dots \quad (A17)$$

the contribution as

$$S_2^{(a')} = -if^2\kappa^4(2\pi)^{-2}\delta(p_3+p_2-p_3-p_4)\kappa_0^2(p_{10}p_{20}p_{30}p_{40})^{-\frac{1}{2}} \\ \times u(p_3)\gamma_5\tau_iu(p_1)\bar{u}(p_4)\gamma_5\tau_iu(p_2)[(A_1-B_1\kappa^2)(q^2+\kappa^2)^{-4}+B_1(q^2+\kappa^2)^{-3}]. \dots \quad (A18)$$

Comparison of Eqns. (A17), (4) and (6) shows that the contributions with the divergent constants A and B are cancelled provided

$$4f^4\kappa^4 A = if^2\kappa^4(2\pi)^4(A_1 - B_1\kappa^2),$$

$$4f^4\kappa^4 B = if^2\kappa^4(2\pi)^4B_1.$$

Remembering substitution (17), the last equation gives the bare masses and the renormalisation terms with A and B given by equations (A3) and (A4). We note that the interpretation of the constants A and B in conventional meson theory went to both mass and coupling constant renormalisation, which is not possible here. Also, two bare masses were needed to interpret two divergent constants as renormalisation terms. On the other hand, if we had started with a bare particle of unique mass, and had considered interaction, the corresponding mass renormalisation would have given rise to a splitting of the mass. This result is interesting, but such a splitting is useless to investigate so long as the equations that determine the splitting, and thus the physical masses, contain infinite constants.

A C K N O W L E D G M E N T

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ULTRAVIOLET ABSORPTION SPECTRA OF ISOMERIC FLUOROTOLUENES IN THE LIQUID AND SOLID STATES*

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ABSTRACT. The ultraviolet absorption spectra of isomeric fluorotoluenes in the liquid and solid states have been analysed and the results have been compared with those for the substances in the vapour state reported by previous workers.

The spectrum due to liquid state of *o*-fluorotoluene exhibits broad bands, the 0,0 band being shifted towards longer wavelengths by 238 cm^{-1} with respect to that due to vapour. The bands become sharper and shift towards shorter wavelengths when the liquid is solidified and cooled to -180°C .

The spectrum due to the liquid *m*-fluorotoluene appears to consist of two series of broad bands which have been assigned to monomers and dimers respectively. Only one system of bands is given by the substance in the solid state at -180°C . It has been shown that these results can be explained if it is assumed that all the molecules are transformed into dimers in the solid state at -180°C .

The spectrum due to *p*-fluorotoluene in the solid state at -180°C seems to consist of two series of bands. It has been possible to analyse them on the supposition that the excited electronic energy level is split up into two components. The shift of the 0,0 band on liquefaction is observed to be larger in the case of the para compound than in the case of the ortho compound. It has therefore been concluded that the larger shift may be due to the formation of dimers even in the liquid state. The cause of the splitting has been discussed.

INTRODUCTION

Investigations on the influence of intermolecular field on the absorption spectra of some halogen substituted benzene compounds in the state of aggregation at low temperature by Swamy (1952, 1953) revealed some interesting facts. Splitting of the electronic energy level into three components was observed in the case of *o*- and *m*-chlorotoluene, *o*-dichlorobenzene and *o*-bromotoluene when the substances were solidified and cooled to -180°C . No splitting was, however, observed in the case of the para compounds. Similar changes in the spectra were observed later in the case of *m*-bromotoluene and *m*-dichlorobenzene by the present author (Sen, 1957). Thus, unsymmetrical disubstituted benzenes containing halogen atoms as substituents showed the splitting of the energy level.

*Communicated by Prof. S. C. Sirkar.

The electronic absorption spectra of free molecules of isomeric fluorotoluenes in the vapour state have been investigated by Cave and Thompson (1950) and the spectra of solutions of isomeric fluorotoluenes in *n*-heptane have been reported by Miller and Thompson (1949). They observed higher extinction co-efficient for *p*-fluorotoluene than in the case of *o*- and *m*-fluorotoluene.

Previous investigations on the influence of intermolecular field on the absorption spectra of halogen substituted benzenes have been confined only to compounds having either chlorine or bromine atom as a substituent. As the fluorine atom is more strongly active than the chlorine or bromine atom, it was thought worthwhile to extend the investigations to isomeric fluorotoluenes. With this object, the ultraviolet absorption spectra of *o*-, *m*- and *p*-fluorotoluene in the liquid state and in the solid state at -180°C have been studied in the present investigation. The results have been discussed and compared with those for the other halogen substituted benzene compounds in the states of aggregation reported by previous workers.

EXPERIMENTAL

Chemically pure samples of *o*-, *m*- and *p*-fluorotoluene (Fisher) were distilled repeatedly under reduced pressure before being used in the present investigation.

The experimental set up was the same as in an earlier investigation by the author (Sen, 1955). Spectrograms were taken on Ilford HP3 films with Hilger El quartz spectrograph having a dispersion of 3 A.U. per mm. in the region of 2600 Å. Very thin films of thicknesses of the order of a few microns were required to produce the bands due to the liquid state and the time of exposure varied from 10 to 15 minutes. The corresponding spectra due to the substances in the frozen state at low temperature were recorded with a time of exposure of about 1 hr. Iron arc spectrum was taken in each spectrogram as comparison. Microphotometric records were obtained with a self-recording microphotometer of Kipp and Zonen type. The frequencies of the bands were measured using the microphotometric records of iron lines as in an earlier investigation (Sen, 1956). The slit width was 0.3 mm ($\equiv 15 \text{ cm}^{-1}$) in the region of 2600 Å.

RESULTS

The microphotometric records of the spectra are reproduced in Figs. 1, 2 and 3 and the wave numbers of the bands with approximate intensities and probable assignments are given in Tables I—III.

DISCUSSION

(a) *o*-Fluorotoluene ($\text{C}_6\text{H}_4\text{CH}_3\text{F}$):

The absorption spectrum of *o*-fluorotoluene in the vapour state had earlier been photographed by Cave and Thompson (1950). The 0,0 band was taken at

37576 cm^{-1} . The rest of the bands were explained as due to progressions of excited state vibration frequencies 707, 924, 1230 cm^{-1} and their combinations.

They pointed out that on the longer wavelength side of the 0,0 band there is a very strong band, but they did not explain why the feebler band was taken as the 0,0 band. On examining the spectrogram reproduced by them, it is found that if the 0,0 band be identified with the strong band at 37484 cm^{-1} , two very sharp and weak bands at 37414 cm^{-1} and 37554 cm^{-1} yield a vibration frequency 70 cm^{-1} both in the ground and excited states. These two frequencies seem to be coupled with all other stronger bands giving progressions of frequencies 707 cm^{-1} , 804 cm^{-1} , 929 cm^{-1} and 1230 cm^{-1} . Also, a band giving an excited frequency 498 cm^{-1} is observed in the spectrum due to the vapour state and this band is observed in the spectrum due to crystals at -180°C . Hence the 0,0 band due to the vapour is fixed at 37484 cm^{-1} .

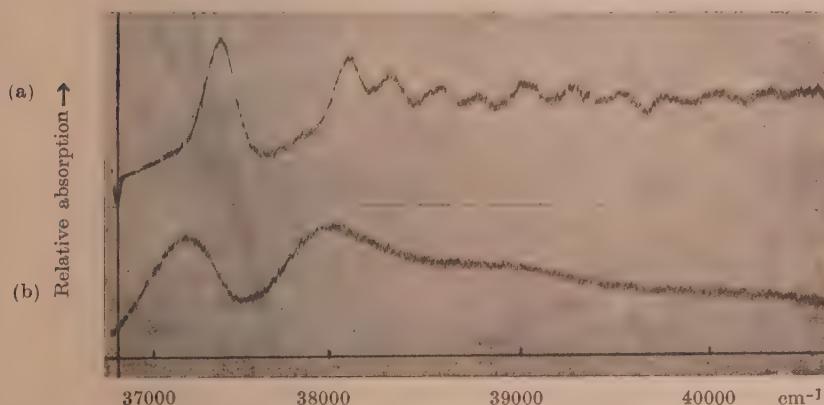


Fig. 1. Microphotometric records of the ultraviolet absorption spectra of *o*-fluorotoluene.
 (a) Solid at -180°C . (b) Liquid at 30°C .

Four broad bands have been observed in the spectrum due to liquid state, the centre of the 0,0 band being at 37246 cm^{-1} . The 0,0 band thus shifts towards longer wavelengths by 238 cm^{-1} with liquefaction of vapour. Such a shift may be explained on the supposition that intermolecular field acting on the molecules in the liquid state lowers the excited electronic energy state. The broadness of the bands may be due to fluctuation of intermolecular field by thermal motion of molecules. The succeeding bands represent progressions of excited state vibration frequencies 707 cm^{-1} and 924 cm^{-1} , which are not different from the corresponding frequencies observed in the vapour state.

TABLE I

Absorption bands of *o*-fluorotoluene in the liquid and solid states

Vapour Cave and Thompson (1950) (Prominent bands)	Present author on the basis of spectrogram repro- duced by Cave & Thompson (1950) (Prominent bands)		Liquid at 30°C.	Solid at -180°C.				
	ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment
36827	0-749	37351(vw)	0-133					
37302	0-274	37414(w, very sharp)	0-70					
37576	0,0	37484(vs)	0,0	37246(s,b)	0,0	37400(vs)	0,0	
38283	0+707	37554(w, very sharp)	0+70	37953(s,b)	0+707	37896(m)	0+496	
38500	0+924	37982(w)	0+498	38170(m,b)	0+924	38107(s)	0+707	
38806	0+1230	38121(w, very sharp)	0+707 -70	38877(w,b)	0+707+924	38324(s)	0+924	
		38191(s)	0+707			38630(m)	0+1230	
		38260(w, very sharp)	0+707+70			38815(m)	0+707×2	
		38218(w, very sharp)	0+804-70			39035(m)	0+707+924	
		38288(s)	0+804			39127(m)	0+496+1230	
		38358(w, very sharp)	0+804+70			39250(m)	0+924×2	
		38413(s)	0+929			39552(m)	0+924+1230	
		38714(m)	0+1230			39741(m)	0+707×2+924	
						40044(m)	0+707×2+1230	
						40261(w)	0+707+924+1230	

On solidification and cooling of the substance to -180°C, the spectrum yields a sharper system of bands with the 0,0 band at 37400 cm^{-1} shifted by 154 cm^{-1} towards shorter wavelengths with respect to its position in the liquid state. Progressions of excited state vibration frequencies $496, 707, 924, 1230\text{ cm}^{-1}$ have been observed. As mentioned earlier, the frequency 496 cm^{-1} also appears in the vapour state and it may correspond to the ground state vibration frequency 530 cm^{-1} . *o*-Fluorotoluene does not seem to exhibit any splitting of electronic energy level unlike orthochloro- and orthobromotoluene.

(b) *m*-Fluorotoluene ($C_6H_4CH_3F$)

The ultraviolet absorption spectrum due to vapour state of *m*-fluorotoluene was analysed by Cave and Thompson (1950). They found a large number of feeble bands within a distance of 240 cm^{-1} from the 0,0 band at 37398 cm^{-1} on the shorter wavelength side and a set of strong bands on the longer wavelength side. The latter bands were assigned to $v \rightarrow 0$ transitions with ground state vibration frequencies 70, 184, 253, 311, 520 and 726 cm^{-1} .

In the liquid state the first band appears to consist of two unresolved broad bands with centres at 36965 cm^{-1} and 37212 cm^{-1} . The next two prominent bands have a separation of 685 cm^{-1} from the first two bands respectively. The succeeding bands are very broad having no structure.

TABLE II

Absorption bands of *m*-fluorotoluene in the liquid and solid states

Vapour Cave & Thompson (1950) (Prominent bands)		Liquid at 30°C		Solid at -180°C .	
ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment
36935	0-1003				
36672	0-726				
36878	0-520				
37087	0-311				
37145	0-184-70				
37176	0-253				
37214	0-184				
37243					
37273	0--70				
37325					
37346					
37368					
37398	0,0	36965 (s)	A_o	37239 (s)	0,0
37486	0+684-520	37212 (s)	B_o	37924 (s)	0+685
37496	-70	37650 (s)	A_o+685	38163 (s)	0+924
37535		37896 (s)	B_o+685	38485 (s)	0+1246
37558	0+684-520				
37573				38853 (m)	0+685+924
37631	0+965-726			39087 (m)	$0+924 \times 2$
37680	0+282			39412 (m)	$0+924+1246$
37892	0+457			39733 (w)	$0+1246 \times 2$
38082	0+684			40331 (w)	$0+924 \times 2+1246$
38363	0+965				
38659	0+1261				

The spectrum seems to consist of two series of bands separated by 247 cm^{-1} and one of these series seems to disappear with solidification of the liquid. Hence it has been assumed that the two series are produced respectively by the monomers and dimers present in the liquid and that in the solid state at -180°C , all the molecules are transformed into dimers.

When the liquid is solidified and cooled to -180°C , the first band becomes narrower and appears to consist of only one broad band with its centre at 37239 cm^{-1} . However, the $0,0$ band due to the frozen mass at -180°C is not sharp but it has a width of about 100 cm^{-1} . This cannot be due to the presence of unresolved bands due to $0 \rightarrow \sigma$ transitions by the side of the $0,0$ band on the shorter wavelength side because the first band is quite symmetrical throughout the whole width, and similarly, all the succeeding bands on the short wavelength side are equally wide. It appears, therefore, that these widths of the bands are due to perturbation produced by neighbouring molecules in the crystal on the electronic energy level of the molecule. This perturbation being small, there is no observable splitting into three components as observed in other cases of halogen substituted meta compounds. It appears from Fig. 2(a) and Table II, that in the spec-

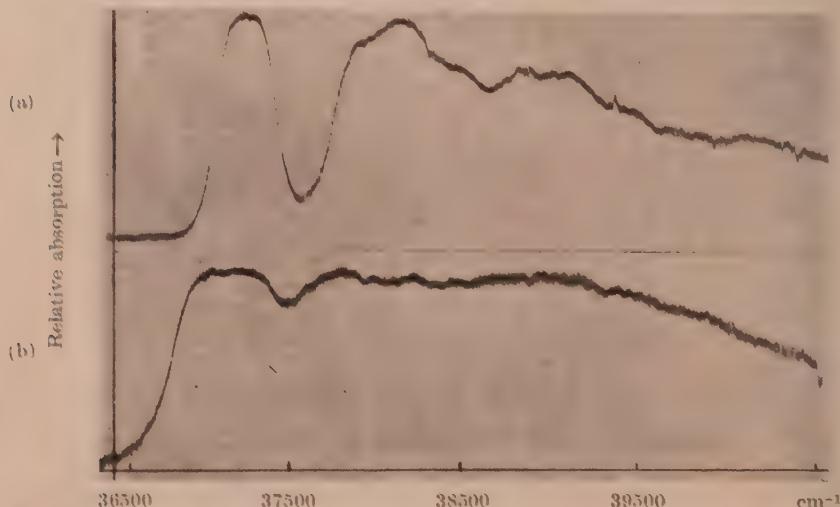


Fig. 2. Microphotometric records of the ultraviolet absorption spectra of *m*-fluorotoluene.
(a) Solid at -180°C . (b) Liquid at 30°C .

trum due to solidified *m*-fluorotoluene at -180°C , there are bands due to vibrational transitions of excited state frequencies 685 , 924 and 1246 cm^{-1} . While the first frequency remains unaltered with change of state, the latter two correspond to the frequencies 965 and 1261 cm^{-1} observed in the vapour state. The fact that the first band due to solid state at -180°C is displaced towards shorter wavelengths from the position of the first band due to the liquid state confirms the assignment

indicated above, because generally with solidification and cooling of the crystals to -180°C . the 0,0 band shifts towards shorter wavelengths. In the present case the 0,0 band at 36965 cm^{-1} due to the dimers shifts towards shorter wavelengths with solidification of the liquid.

The assignment of the bands on both sides of the 0,0 band due to vapour state made by Cave and Thompson (1950) is confirmed by the fact that these bands disappear at low temperature.

(c) *p*-Fluorotoluene ($\text{C}_6\text{H}_4\text{CH}_3\text{F}$)

The analysis of the spectrum due to para compound in the vapour state made by Cave and Thompson (1950) shows the 0,0 band to be at 36876 cm^{-1} and the succeeding bands are assigned to progressions of excited state frequencies 185, 398, 584, 794, 843, 1014, 1194 and 1229 cm^{-1} . In the liquid state broad bands have been observed with the 0,0 band at 36517 cm^{-1} . Thus, there is a shift of the 0,0 band by 359 cm^{-1} towards longer wavelengths on liquefaction of vapour. The succeeding bands represent transitions corresponding to vibrational frequencies 843 and 1230 cm^{-1} not different from the corresponding frequencies observed for the vapour state.

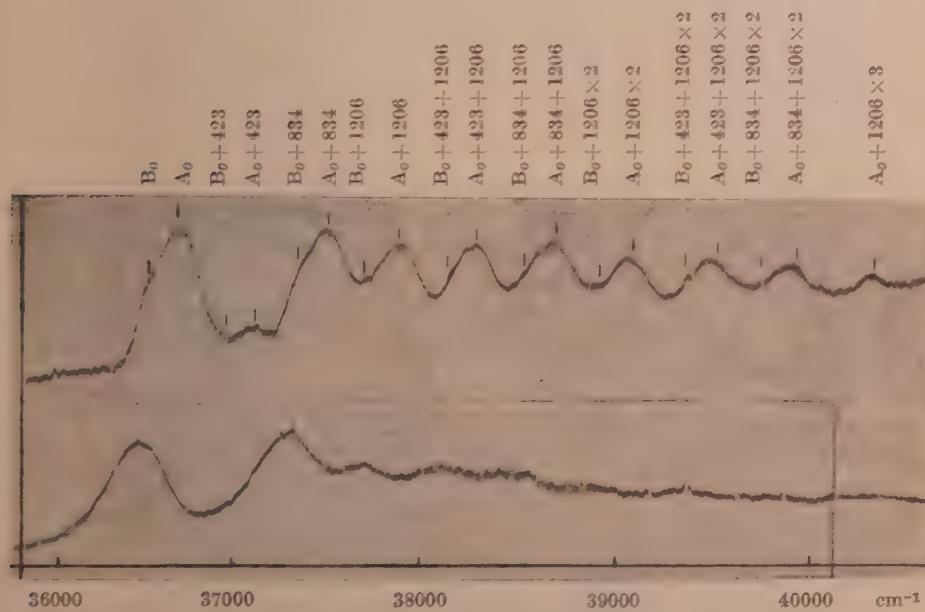


Fig. 3. Microphtometric records of the ultraviolet absorption spectra of *p*-fluorotoluene.

(a) Solid at -180°C .

(b) Liquid at 30°C .

It is evident from Fig. 3(a) and Table III that in the spectrum due to solid state at -180°C , each strong band is accompanied by a weaker one on the longer wavelength side. The first weak band cannot be due to a $v \rightarrow 0$ transition, because the corresponding band due to $0 \rightarrow v$ transition is absent. Hence, it is concluded that there is a splitting of the bands in the solid state. The distance between the components is observed to be 188 cm^{-1} and assignments of the bands have been made on the assumption that the excited electronic energy level is split up into two components with a separation of 188 cm^{-1} .

The excited state vibrational frequencies are observed to be 423 , 834 and 1206 cm^{-1} . The new excited state frequency 423 cm^{-1} does not seem to have been observed in the vapour state and the other two frequencies may correspond to the frequencies 843 cm^{-1} and 1229 cm^{-1} observed in the spectrum due to the vapour. The intense component of the $0,0$ band is shifted towards shorter wavelengths by 223 cm^{-1} .

A comparison of the spectra due to the vapours of the ortho- and para dichlorobenzene shows that the $0,0$ band shifts to longer wave lengths by about 450 cm^{-1} with the change from the ortho to the para configuration. In the present case a similar shift of about 608 cm^{-1} is observed with the change from ortho to the para position of the fluorine atom. When the vapour is liquefied the $0,0$ band again shifts towards longer wavelengths by about 359 cm^{-1} while in the case of the ortho compound the shift with liquefaction of vapour is only 238 cm^{-1} . Hence this larger shift in the case of the para compound may be due to formation of dimers in the liquid state.

In the case of *o*-chlorotoluene, *o*- dichlorobenzene and *m*-dichlorobenzene (Swamy, 1952, 1953; Sen, 1957) each band was found to be split up into three components, the central component being the strongest. In the spectra of *m*-chlorotoluene (Swamy, 1952), *o*-bromotoluene (Swamy, 1953) and *m*-bromotoluene (Sen, 1957) in the solid state the strongest of the three components is on the short wavelength side of the other two components. In the case of *p*-fluorotoluene also, the weaker component is on the longer wavelength side of the stronger component in each of the bands.

It has already been pointed out by Sirkar and Misra (1959) that the so called Davydov splitting is not observed in the case of certain crystals owing probably to the fact that the contribution of only a few neighbours predominates in the interaction of transition moments of neighbouring molecules on the energy of any particular molecule, and therefore, the oscillator strength determined with a large number of molecules in the absorbing path cannot be used in this case. They also pointed out that when the transition takes place in the field of the permanent dipoles surrounding any molecule in the crystal there may be a splitting of the bands. Probably in the case of *p*-fluorotoluene also, the splitting is due to the interaction of such dipoles on the electronic transition in the molecule.

TABLE III

Absorption bands of *p*-fluorotoluene in the liquid and solid states

Vapour Cave & Thompson (1950) (Prominent bands)		Liquid at 30°C.		Solid at -180°C	
ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment	ν in cm^{-1}	Assignment
35720	0-1156				
36632	0-844				
36051	0-825				
36235	0-641				
36423	0-453				
36539	0-337				
36565	0-311				
36659	0-217				
36780	0-96				
36831	0-40			36552 (w)	B_o
36876	0,0	36517 (s,b)	0,0	36740 (s)	A_o
36966	0+185-96	37360 (s,b)	0+843	36975 (w)	B_o+423
37004	0+584-453	37747 (m,b)	0+1230	37163 (m)	A_o+423
37061	0+185	38200 (m,b)	0+843 \times 2	37386 (m)	B_o+834
37175	0+398-96	38591 (w,b)	0+843+1230	37568 (s)	A_o+834
37218	0+398-40	39435 (w,b)	0+843 \times 2	37758 (w)	B_o+1206
37231			+1230		
				37948 (s)	A_o+1206
37274	0+398			38185 (w)	$B_o+423+1206$
37460	0+584			38368 (s)	$A_o+423+1206$
37670	0+794			38598 (w)	$B_o+834+1206$
	0+398 \times 2				
37719	0+843			38778 (s)	$A_o+834+1206$
37890	0+1014			38968 (w)	$B_o+1206 \times 2$
38070	0+1194			39157 (s)	$A_o+1206 \times 2$
38105	0+1229			39389 (w)	$B_o+423+1206 \times 2$
				39576 (s)	$A_o+423+1206 \times 2$
				39796 (w)	$B_o+834+1206 \times 2$
				39989 (s)	$A_o+834+1206 \times 2$
				40359 (m)	$A_o+1206 \times 3$

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RAMAN SPECTRA OF ORTHO- AND PARACHLOROANISOLE IN THE SOLID STATE AT -180°C^*

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Plate III

ABSTRACT. The Raman spectra of ortho- and parachloroanisole have been studied in the liquid state and in the solid state at -180°C . It has been observed that in the case of parachloroanisole the components of each of the doublets 625 and 638 cm^{-1} and 1584 and 1596 cm^{-1} coalesce to form a single line of Raman shift 640 cm^{-1} in the former case and 1603 cm^{-1} in the latter case. When the liquid is solidified a new line 3026 cm^{-1} also appears in the spectrum. Besides these changes three new low frequency Raman lines 60 , 92 and 132 cm^{-1} also appear with solidification of the liquid.

In the case of orthochloroanisole no appreciable changes take place in the spectrum and no new low frequency lines appear with the solidification of the liquid. It has been concluded from these results that in the case of the ortho compound the OCH_3 group is probably chelated to the adjacent chlorine atom and therefore the formation of associated groups is not possible in the solid state. Such groups are, however, formed when the para compound is solidified and the new low frequency lines observed in this case are attributed to such groups of molecules.

INTRODUCTION

Since the discovery of new low-frequency Raman lines in the Raman spectra of some substituted benzene compounds in the solid state by Gross and Vuks (1936) many workers have investigated the Raman spectra of a large number of such solidified benzene compounds at different temperatures (Sirkar, 1937; Mazumder, 1949; Ray, 1950a,b, 1951, 1952; Biswas, 1954a,b,c, 1955a,b,c, 1956a; and others). It has been concluded by these workers from the results obtained by them that the low-frequency Raman lines originate from vibrations in small groups of molecules formed by the attachment of the individual molecules to neighbouring molecules through weak linkages and that these lines cannot be attributed either to angular oscillations of the molecules about their axes as suggested by Kastler and Rousset (1941) and by Bhagavantam (1941) or to translational lattice oscillation as suggested by Gross and Vuks (1936). Generally, halogen substituted benzene compounds yield strong low-frequency Raman lines in the solid state and it has been pointed out that the formation of weak intermolecular bonds is more probable in such cases than in the molecules in which no such halogen atom

*Communicated by Prof. S. C. Sirkar.

is present as a substituent. In certain disubstituted benzene compounds, such as chlorophenol, chloroanisole, etc., there is rotational freedom of the OH or OCH_3 group about the C-O bond, but the relative positions of the halogen atom and the other substituent have influence on such rotational freedom. It would be interesting to find out whether the formation of intermolecular linkages in the solid state and the consequent production of low-frequency Raman lines are influenced by relative positions of the two substituents in such cases. As a preliminary investigation of this problem, the Raman spectra of ortho- and parachloroanisole in the solid state at -180°C have been studied and the results have been compared with those for the liquids.

EXPERIMENTAL

The liquids were of chemically pure quality supplied by British Drug House. They were further purified by distillation under reduced pressure. While recording the spectra due to the liquids a strong continuous fluorescence in the visible region was observed. Hence, to reduce the intensity of this fluorescence a second photograph of the spectrum was taken in each case using Rhodamine 6 GBN filters which cut off 4046 Å group of mercury lines. The fluorescence was found to be stronger in the case of orthochloroanisole than in the case of the para compound.

In order to record the Raman spectra of the compounds in the solid state attempt was made to obtain a homogeneous solid mass by cooling the liquid slowly so that the stray light due to diffuse reflection from the inner portion of the solid could be reduced considerably. The experimental arrangement in this case was the same as that used by Biswas (1954). The Raman spectra were recorded on Ilford Zenith plates using a Fuess glass spectrograph giving a dispersion of about 11 Å in the region of 4046 Å. The polarisation of the Raman lines due to each of the compounds was also studied in the usual manner, but in the case of the ortho compound a freshly prepared 5% NaNO_2 solution was used as a filter in order to avoid the continuous fluorescence in the regions from 4358 Å to 4916 Å.

RESULTS AND DISCUSSION

The spectrograms are reproduced in Plate III. The Raman frequencies of the lines are given in Tables I and II. The frequencies reported for the liquids by previous workers are also included in the tables. The states of polarisation of the Raman lines due to the compounds are indicated in the respective tables by the usual symbols $D(\rho \geq 6/7)$ and $P(\rho < 6/7)$, ρ being the depolarisation factor.

(a) *Parachloroanisole* :

It can be seen from Table I and Figs. 1(a) and 1(b) that the following changes take place when this liquid is solidified and cooled to -180°C .

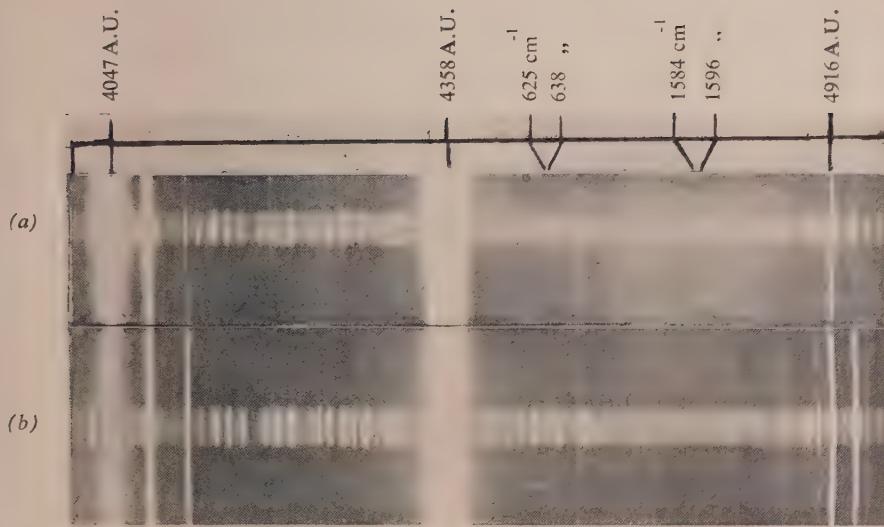


Fig. 1

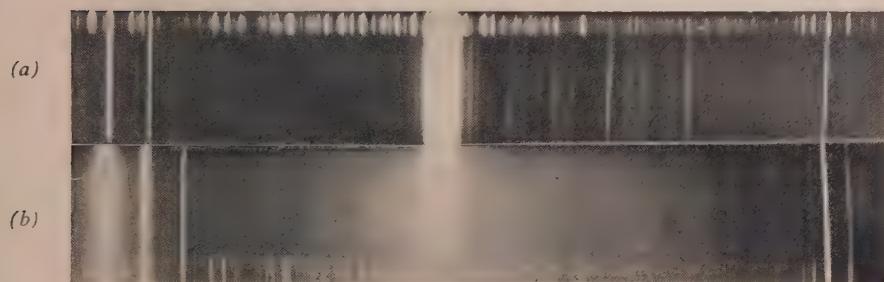


Fig. 2

Fig. 1. (a) Raman spectra of parachloroanisole liquid at 30°C

(b) " " parachloroanisole solid at -180°C

Fig. 2. (a) " " orthochloroanisole liquid at 30°C

(b) " " orthochloroanisole solid at -180°C

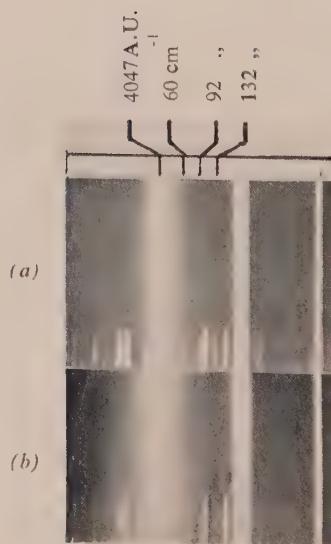


Fig. 3

Fig. 3. (a) Raman spectra of parachloroanisole solid at -180°C , showing low-frequency lines.

(b) " " orthochloroanisole solid at -180°C

Three new Raman lines with Raman shifts 60, 92 and 132 cm^{-1} appear in the spectra due to the solid (Fig. 3a), the line 92 cm^{-1} being the strongest. Further, some of the prominent Raman lines of the molecule undergo changes in position and intensity with the solidification. The intensity of the doublet 625 cm^{-1} and 638 cm^{-1} is reduced appreciably and the two lines coalesce to form a single line 640 cm^{-1} . The line 799 cm^{-1} shifts to 804 cm^{-1} and the lines 1584 cm^{-1} and 1596 cm^{-1} coalesce to form a single line at 1603 cm^{-1} . Finally, a new strong line at 3026 cm^{-1} appears in the spectrum due to the solid.

The diminution in the number of Raman lines with the solidification of the liquid may be due to the fact that there exist in the liquid state two types

TABLE I
Parachloroanisole. $\Delta\nu$ in cm^{-1}

Liquid		Solid at -180°C
Hertz (1947)	Present author	Present author
		60 (3)
		92 (8b)
		132 (3)
143 (3) ?	150 (0) ?	145 (3)
212 (3)	218 (1b) P	225 (0)
309 (4)	313 (3) P	313 (2)
335 (4)	335 (4) D	335 (1)
366 (5)	367 (4) P	367 (2)
498 (1)		
625 (5)	625 (5) P	
636 (5)	638 (3b) P	640 (1)
698 (3)	695 (0) D	695 (0)
797 (10)	799 (10) P	804 (10)
829 (1)	826 (0) P	
1006 (3)	1005 (1) P	1005 (1)
1034 (1)	1035 (0) P	
1091 (8)	1091 (8) P	1094 (3)
1100 (1)		
1168 (3)	1170 (4b) P	1169 (2)
1181 (3)	1182 (3) P	1188 (1)
1244 (3)	1243 (3) P	1250 (0)
1292 (3)	1293 (4b) P	1299 (2)
1404 (1)		
1440 (3)	1443 (2) P	
1458 (3)	1459 (3b) P	1458 (1)
1492 (1)		
1580 (4)	1584 (3) D	
1594 (7)	1596 (6b) D	1603 (5)
2836 (4)	2844 (4) P	2844 (2)
2937 (2)	2938 (2) P	2838 (1)
3010 (1)		
3064 (6b)	3064 (4b) P	3064 (5)
3075 (6b)	3073 (5) P	3073 (4)

of molecules, namely, monomers and dimers, the latter being formed through weak intermolecular bonds between two neighbouring molecules. When the liquid is solidified probably all the molecules are transformed into small groups of molecules formed by association of the monomers and dimers through virtual linkages. The fact that the lines 625 cm^{-1} and 638 cm^{-1} change to a single line 640 cm^{-1} with the solidification of the liquid shows that these virtual linkages take place between Cl and H atoms of the two neighbouring molecules respectively. As an example of such small splitting of some of the Raman lines with formation of dimers, mention can be made of the change in the Raman spectra of the solution of pyridine with dissolution in alcohol (Kastha, 1956). The spectra due to pure pyridine gives two Raman lines at 995 cm^{-1} and 1029 cm^{-1} of almost the same intensity, but when the liquid is dissolved in alcohol the former line becomes very weak. This is due to the fact that the non-bonding electron of the N-atom of pyridine forms a bond with the OH-group of the neighbouring alcohol molecules. In the present case also of the two lines 625 cm^{-1} and 638 cm^{-1} , the first one may be due to the monomers and the second one due to the dimers, and with solidification the former line disappears owing to formation of groups of molecules. It is also found that a new line 3026 cm^{-1} is produced when the liquid is solidified. This shows that probably the H-atom attached to the benzene ring is responsible for the formation of virtual linkages with the Cl-atom of the neighbouring molecules. The disappearance of the line 1584 cm^{-1} also leads to the conclusion that this line is due to monomers while the other line 1596 cm^{-1} which persists in the solid state is due to the dimers.

(b) *Orthochloroanisole* :

On examination of the Raman spectra of orthochloroanisole in the liquid and solid states (Figs. 2a and 2b) it is found that no appreciable change takes place in the spectrum when the liquid is solidified. Hence, the formation of dimers in the liquid is not observed in this case. This may be due to the fact that the Cl-atom, which is the reactive atom for the formation of the dimers, is chelated to the OCH_3 group, and therefore, it is no longer able to form a virtual bond with neighbouring molecules.

Tables I and II and also Figs. 3a and 3b show that *p*-chloroanisole produces three strong new low-frequency lines 60 , 92 and 132 cm^{-1} in the solid state at $\sim 180^\circ\text{C}$ while the ortho compound under the same condition does not produce any new line in this region. It has already been pointed out that in the para compound groups of molecules are formed while in the case of the ortho compound even in the solid state all the molecules remain as monomers. These facts show that the new lines in the low frequency region produced by the para compound in the solid state are due to groups of molecules. Had these lines been due to lattice vibrations they would also be produced by the ortho compound. The

TABLE II
Orthochloroanisole. $\Delta\nu$ in cm^{-1}

	Liquid	Solid at -180°C	
	Hertz (1946)	Present author	Present author
160 (5b)	160 (6b) D	165 (2b)	
204 (3b)	202 (4) D	202 (2b)	
288 (1)	290 (2vb) P		
410 (5b)	407 (8) P	407 (2)	
444 (1)	440 (0)		
493 (6)	493 (8) P	493 (3)	
546 (0)	550 (0) P		
576 (1b)	573 (3) D	573 (0b)	
685 (8)	687 (8) P	692 (6)	
712 (1)	708 (0) D		
753 (1b)	756 (1b) D		
785 (1)			
796 (7)	798 (6) P	798 (8)	
844 (0)			
925 (0)			
995 (2)	995 (1)		
1025 (2)	1025 (1) P		
1041 (10)	1038 (10) P	1038 (8)	
1065 (1)			
1092 (1)	1091 (0)		
1133 (1)	1130 (1)		
1162 (5)	1164 (6) D	1164 (1b)	
1183 (3)	1181 (2d) P	1180 (0)	
1208 (0)			
1250 (6)	1253 (5) P	1257 (2)	
1274 (2)	1271 (2) P		
1300 (3)	1299 (3) P	1302 (1)	
1360 (0)			
1436 (1)	1462 (2) D		
1463 (1)			
1486 (2)	1485 (3) P	1484 (1b)	
1576 (5)	1576 (7) D	1576 (3)	
1588 (5)	1588 (7) D	1586 (3)	
2831 (2b)	2833 (3b) P	2833 (0)	
2948 (2)	2948 (2b) P		
3004 (8)			
3068 (8)	3071 (9vb) P	3073 (6)	
3076 (7b)			

results thus lead to the conclusion that the low frequency lines are due to vibrations in groups of molecules.

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BOOK REVIEW

NUCLEAR ELECTRONICS I (SESSIONS 1-5)... pp.452. Being the Proceedings of the International Symposium on Nuclear Electronics organised by the French Society of Radioelectricians held in Paris 1958.

In September, 1958 the French Society of Radioelectricians organised a Colloquium on Nuclear Electronics in Paris. The publication under review is an edited version of the papers and discussions of this Colloquium published by the International Atomic Energy Agency from Vienna. The major part of the papers and discussions in this Colloquium was divided into five sessions, three of them devoted to scintillation counters and to fast counting techniques in the region of milli-micro seconds or less using photomultipliers. The fourth session was devoted mainly to pulse height and time analysers and the fifth to reactor instrumentation and reactor control techniques.

It is interesting to note that papers from many lands and different laboratories show parallel developments in the field of scintillation counting techniques. The papers also reflect the general attitude that one is fast approaching the limit of conventional electronic tubes in speed and pulse amplification. This is demonstrated by the general search for techniques for the elimination of ordinary electronic tubes in the achievement of improved measurements of fast pulses. The study of scintillation processes in many phosphors is also discussed in several papers as a prelude to attempts to use the fast initial component of the light pulse.

The field of pulse height and time analysers covered in the 4th session describes current efforts in this field. The necessity of using fast pulses has given rise to a few techniques different from the conventional pulse height analysers which have proved unsuitable for very fast pulses. The topic is well covered in a preliminary review paper followed by several more or less detailed contributions on different schemes. The last group of papers on reactor instrumentation and control reflect mostly problems encountered in the control and instrumentation of the French Reactors and do not describe the techniques connected with the operation control of the novel fast reactors involving more difficult problems of reactor control.

It is interesting to find contributions on the use of digital computer techniques and standardised transistor circuitry for reactor control. Their greater reliability under radiation had been foreseen earlier and these techniques are likely to play an important role in future reactor instrumentation.

The IAEA venture of publishing proceedings of Symposia is, the reviewer feels, a timely and useful step in the dissemination of technical information and deserves our congratulations. The proceedings are well printed and the illustrations are clear and profuse.

B. D. N. C.

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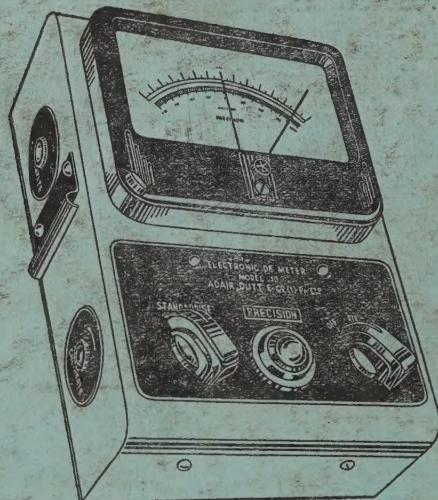
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